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Hamilton Cycles And Spanning 2-Connected Subgraphs In Rectangular Grid Graphs

L. Eswaramma[1] and Dr D. Venkata Lakshmi[2]

[1]Department of Mathematics, Aurora's Technological and Research Institute, Hyderabad, India.

[2]Department of Mathematics, Bapatla Engineering College, AP, India

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Abstract:* Graph theory concepts are widely used to study and model various applications in different areas such as computer graphics, electrical network analysis and operation research. A Hamiltonian cycle in a graph is a cycle that visits each vertex exactly once.** *These concepts helps in routing power gating design to reduce the leakage of power by break the flow of inactive circuit domains.* **A connected graph is 2-connected if at most one vertex is removed the graph remains connected. A spanning 2-connected subgraph is in which every vertex has degree 2. In this paper we studied Hamilton cycles and Spanning 2-connected sub graphs for 3-corner and 4-corner rectangular grid graphs.

***Keywords:* Hamilton cycles, Rectangular grid graphs and Spanning 2- connected subgraph.**

1. INTRODUCTION

Hamilton cycles for 1-corner and 2-corner rectangular grid graphs for some cases were studied in [4].Spanning 2-connected sub graphs for 1-corner and 2-corner rectangular grid graphs for some cases were studied in [5].In this paper certain cases for 3-corner and 4-corner rectangular grid graphs were studied .

We introduce the classes of grid graphs which we call it as truncated rectangular grid graphs.

For $s \geq 3, t \geq 3, 0 \leq k \leq \min\{s-2, t-2\}$ and $0 \leq l \leq \min\{s-2, t-2\}$. we define a 1-corner truncated rectangular grid graph $R(s,t)^{-1(k,l)}$ as the subgraph obtained from $R(s,t)$ by deleting $k \times l$ vertices from one corner in $V(s,t)$ together with their incident edges in a natural drawing. For illustration, consider $R(13,11)^{-1(3,2)}$ in figure.2.1(a).

For $s \geq 6, t \geq 6, 1 \leq k \leq \min\left\{\frac{s-4}{2}, \frac{t-4}{2}\right\}$ and $1 \leq l \leq \min\left\{\frac{s-4}{2}, \frac{t-4}{2}\right\}$ we define a $2-$ corner truncated rectangulargrid graph $R(s,t)^{-2(k,l)}$ as the subgraph obtained from $R(s,t)$ by deleting $k \times l$ vertices from two opposite corners in $V(s,t)$ together with their incident edges in a natural drawing. For illustration, consider $R(13,11)^{-2(3,2)}$ in Figure 2.1 (b).

For $s \geq 6, t \geq 6, 1 \leq k \leq \min\left\{\frac{s-4}{2}, \frac{t-4}{2}\right\}$ and $1 \leq l \leq \min\left\{\frac{s-4}{2}, \frac{t-4}{2}\right\}$ we define a 3-corner truncated rectangular grid graph $R(s,t)^{-3(k,l)}$ as the subgraph obtained from $R(s,t)$ by deleting $k \times l$ vertices from three corners in $V(s,t)$ together with their incident edges in a natural drawing. For illustration, consider $R(13,11)^{-3(3,2)}$ in Figure 2.1 (c)

For $s \geq 6, t \geq 6, 1 \leq k \leq \min\left\{\frac{s-4}{2}, \frac{t-4}{2}\right\}$ and $1 \leq l \leq \min\left\{\frac{s-4}{2}, \frac{t-4}{2}\right\}$ we define a 4-corner truncated rectangular grid graph $R(s,t)^{-4(k,l)}$ as the subgraph obtained from $R(s,t)$ by deleting $k \times l$ vertices from three corners in $V(s,t)$ together with their incident edges in a natural drawing. For illustration, consider $R(13,11)^{-4(3,2)}$ in Figure 2.1 (d).

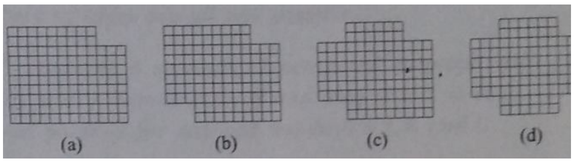

Figure 2.1:Truncated rectangular grid graphs

(a) $R(13,11)^{-1(3,2)}$ (b) $R(13,11)^{-2(3,2)}$ (c) $R(13,11)^{-3(3,2)}$ (d) $R(13,11)^{-4(3,2)}$

2. MAIN RESULTS

Theorem 1:
Let $R(s,t)^{-1(k,l)}, R(s,t)^{-2(k,l)}, R(s,t)^{-3(k,l)} and R(s,t)^{-4(k,l)}$ denote the 1-corner truncated rectangular grid graph ,the 2-corner truncated rectangular grid graph ,the 3-corner rectangular grid graph and 4-corner rectangular grid graph as defined above respectively.Then

a) $R(s,t)^{-1(k,l)}$contains a spanning 2–connected subgraphwith (at most) |V| + 1 edge and is hamiltonian if and only if both s.t and k.1 are even or both s.t and k.1 are odd.

b) $R(s,t)^{-2(k,l)}$contains a spanning 2 – connected subgraphwith

- |V| edges if s.t is even and at least one of k and l is even
- |V| + 2edges if s and t are even and k and l areodd.
- |V| + 1 edges in all othercases.

These number of edges are all best possible.

Discussion: Here we observe certain cases of the above results.

From theorem 1.(a)

Case 1 : The Hamilton cycle for s=8,t=6,k=2and l=2

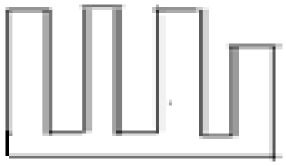

Figure1: $R(8,6)^{-1(2,2)}$

Here both s.t and k.1 are even.

Case2:Spanning 2-connected subgraphfors=9,t=7,k=1 and l=3.

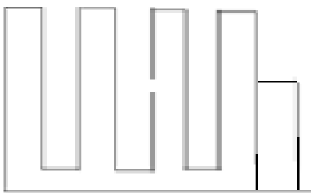

Figure 2: $R(9,7)^{-1(1,3)}$

No. of vertices = 60

No. of edges = 61

Hereboth s.t and k.1 are odd.

Case 3 : From Theorem 1.b

Spanning 2-connected subgraph with |V| + 2edges for s=10,t=8,k=1 and l=3.

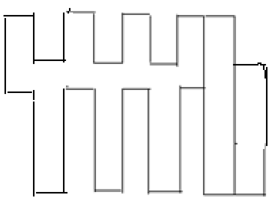

Figure3: $R(10,8)^{-2(1,3)}$

No. of vertices = 74

No. of edges = 76

Here s and t are even and k and l areodd

Theorem2:
Let $R(s,t)^{-3(k,l)} and R(s,t)^{-4(k,l)}$ denote the 3-corner rectangular grid graph and 4-corner rectangular grid graph as defined above respectively.Then

1) $R(s,t)^{-3(k,l)}$containsaspanning2–connectedsubgraph with
- |V| edges if both s.t and k.l areeven;
- |V| + 2 edges if all of s, t, k and l areodd;
- |V| + 1 edges in all other cases. These number of edges are all bestpossible.

2) $R(s,t)^{-4(k,l)}$containsaspanning2–connected subgraphwith (atmost)

- |V| + 3 edges and is hamiltonian if and only if s.t is even. The bound edges and
- |V| + 3 is best possible for any odd numbers s, t, k and l.

Proof:

Proceeding same as in the above theorem

Lemma 1:

Suppose a planar graph G has a Hamilton cycle H. Let G be drawn in the plane, and let r_i denote the number of faces inside H bounded by i edges in this planar embedding . Let r_i^1 be the number of faces outside H bounded by i edges .Then the numbers r_i and r_i'satisfy the following equation.

$$\sum_i (i-2)(r_i - r_i') = 0.$$

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We use this lemma to show that $R(s,t)^{-1(k,l)}$ and $R(s,t)^{-3(k,l)}$ contain no Hamilton cycle if s.t andk.l have different parity,and that $R(s,t)^{-4(k,l)}$ contain no Hamilton cycle if s.t is odd.

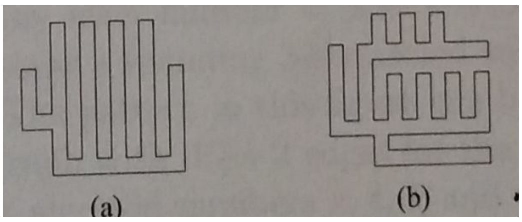
Corollary1.1:

$R(s,t)^{-j(k,l)}$Contains no Hamilton cycle if (s.t and k.l have a different parity for J=1 or 3)or(s.t is odd for j=2 or 4)

Proof was studied in [5].

Discussion: Here we observe certain cases of the above results.

- Hamilton cycles for $R(12,11)^{-3(2,3)}$ and $R(12,11)^{-3(3,2)}$ are as shown in the following Figure .



Case 4 :Using the above pattern(a) Hamilton cycle for s=8,t=10,k=2 and l=3:

Figure 4:$R(8,10)^{-3(2,3)}$

Here s ,k and t are even numbers l is odd number .

Case 5 :Using the above pattern (b) Hamilton cycle for s=10,t=9,k=1 and l=2.

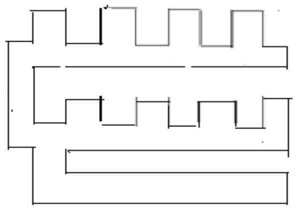


Figure 5:$R(10,9)^{-3(1,2)}$

Here s,l are even numbers and t and k are odd numbers .

Lemma 2: $R(s,t)^{-3(k,l)}$ contains no spanning 2-connected subgraph with at most $|V|+1$ edge if all of s,t,k and l are odd.

Proof : Consider a bipartition (S,T) of $R(s,t)$ for odd s and t. Assume that one of the corner vertices is in S. By the same arguments as in the proof of theorem 1, then all corner vertices are in. S,. And $|S|=|T|+1$. The same holds for $R(k,l)$ if k and l are odd. So if we remove the three corner $R(k,l)'s$ from $R(s,t)$, we reduce $|S|$ by three more units than $|T|$, implying that $R(s,t)^{-3(k,l)}$ has a bipartition (S',T') with $|T'|=|S'|+2$. In any spanning 2-connected subgraph G of $R(s,t)^{-3(k,l)}$ all vertices in T' have degree at least 2, hence $|E(G)|\geq 2|T'|=|T'|+|S'|+2=|V(G)|+2$.

- Spanning 2-connected subgraph for $R(12,10)^{-3(3,3)}$ with $|V|+1$ edges.

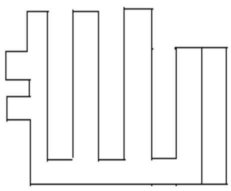


Figure 6

Using the above pattern spanning 2-connected subgraph for s=8,t=10,k=1 and l=3.

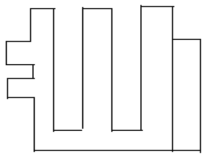


Figure 6.1: $R(8,10)^{-3(1,3)}$

No.of Vertices =71

No.of edges =72

Here s and tare even numbers ,and k and l are any odd numbers .

- Spanning 2-connected subgraph for $R(13,11)^{-3(2,2)}$ with $|V|+1$ edges.

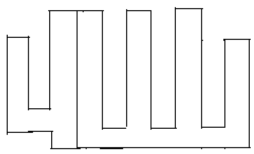


Figure 7

Using the above pattern spanning 2-connected subgraph for s=9,t=7,k=2 and l=2.

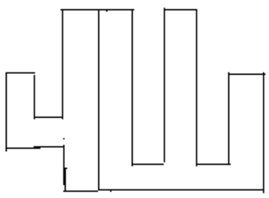


Figure 7.1: $R(9,7)^{-3(2,2)}$

No.of Vertices =51

No.of edges =52

Here k and lare even numbers ,and s and t are odd numbers .

- Spanning 2-connected subgraph for $R(13,11)^{-3(3,2)}$ with $|V|+1$ edges.

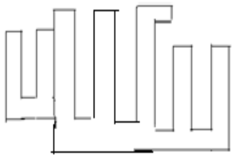


Figure 8

Using the above pattern spanning 2-connected subgraph fors=11,t=9,k=3 and l=2.

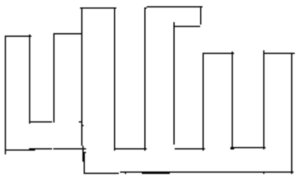


Figure 8.1 : $R(11,9)^{-3(3,2)}$

No.of Vertices =81

No.of edges =82

Here l is even number and any s and t and k are

odd numbers .

- Spanning 2-connected subgraph for $R(13,11)^{-3(3,3)}$ with $|V|+2$ edges.

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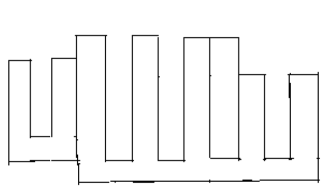

Figure 9

Using the above pattern spanning 2-connected subgraph for s=9,t=7 ,k=3 and l=1.

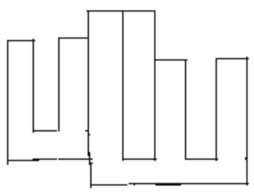

Figure 9.1: $R(9,7)^{-3(3,1)}$

No.of Vertices =54

No.of edges =56

Here s, t, k and l are odd numbers .

Lemma 3: $R(s,t)^{-4(k,l)}$ contains no spanning 2-connected subgraph with at most $|V| + 2$ edges if all of s, t, k and l are odd.

Proof: Consider a bipartition (S,T) of $R(s,t)$ for odd $s$and t. Assume that one of the corner vertices is in S. By the same arguments as in the proof of theorem 1, then all corner vertices are in S, and $|S| = |T| + 1$. The same holds for $R(k,l)$ if k and l are odd. So if we remove the four corner $R(k,l)'s$from $R(s,t)$, we reduce $|S|$ by four more units than $|T|$, implying that $R(s,t)^{-4(k,l)}$ has a bipartition (S', T') with $|T'| = |S'| + 3$. In any spanning 2-connected subgraph G of $R(s,t)^{-4(k,l)}$ all vertices in T' have degree at least 2, hence $|E(G)| \geq 2|T'| = |T'| + |S'| + 3 = |V(G)| + 3$.

We complete the proof of theorem by showing through construction the existence of spanning 2-connected subgraph with atmost $|V| + 3$ edges.

- Spanning 2-connected subgraph for $R(13,11)^{-4(3,3)}$ with $|V|$+3 edges.

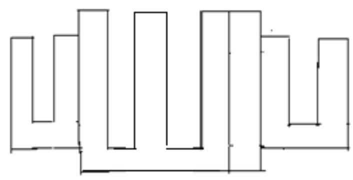

Figure 10

Using the above pattern spanning 2-connected subgraph with $|V|$ +3 edges for s=11,t=9,k=3 and l=1.

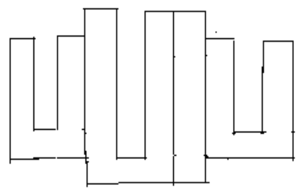

Figure 10.1: $R(11,9)^{-4(3,1)}$

No.of Vertices =87

No.of edges =90

Here s, t, k and l are odd numbers

- Hamilton cycles for $R(12,11)^{-4(2,3)}$ as shown in the following figure .

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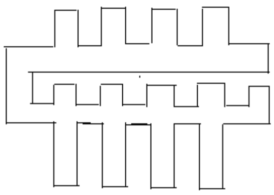

Figure11

Using the above pattern the Hamilton cycle for s=8,t=7,k=2 and l=1.

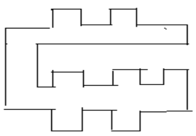

Figure11.1 : $R(8,7)^{-4(2,1)}$

Here s and k are even numbers,t and l are odd umbers

- Hamilton cycle for $R(12,11)^{-4(3,2)}$ as shown in the figure.

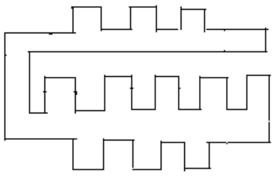

Figure 12

Using the above pattern the Hamilton cycle for s=10,t=8,k=3 and l= 1 .

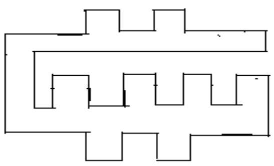

Figure 12.1 : $R(10,8)^{-4(3,1)}$

Here t and l are any numbers, s is even number and k is odd number .

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