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Doubt Intuitionistic Fuzzy Medial K-ideals in BCI-algebras

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Abstract

In this paper we introduce the notion of Doubt Intuitionistic Fuzzy Medial K-ideals in BCI-algebras and investigate some interesting results. Finally we prove that every doubt intuitionistic fuzzy medial K-ideal of X is doubt intuitionistic fuzzy subalgebra of X.

Keywords : medial BCI algebras, fuzzy medial ideals in BCI algebras, intuitionistic fuzzy medial ideal, medial K-ideal, Doubt intuitionistic fuzzy medial K-ideal.

I. INTRODUCTION

The concept of BCK-algebra was introduced by Imai and Iseki in 1966 and the idea of a BCI-algebra was launched by Iseki in the same year which is a expansion of a BCK-algebra. Xi practiced the notion of a fuzzy set to BCK-algebra and established fuzzy sub-algebra and fuzzy ideals in BCK-algebra. The notion of IFS was first established by Atanassov, as a extension of the concept of fuzzy sets. J.Meng and Y.B.Jun studied medial BCI-algebras. Here in this paper, we modify the ideas of Atanassov and Jun to introduce the notion of doubt intuitionistic fuzzy medial K-ideals in BCI-algebras and obtain some interesting results.

II. PRELIMINARIES

Definition 2.1 A BCI-algebra $(X,*,0)$ of type (2,0) is called a medial BCI-algebra if

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it satisfying the following condition : $(x * y) * (z * u) = (x * z) * (y * u)$

$\forall\, x, y, z, u \in X$

Lemma 2.2 An algebra $(X, *, 0)$ of type (2,0) is a medial BCI-algebra if and only if it satisfies the following conditions

$i)\quad x * (y * z) = z * (y * x)$

$ii)\quad x * 0 = x$

$iii)\quad x * x = 0$

Lemma 2.3 In a medial BCI-algebra X, the following holds : $x * (x * y) = y$
$\forall\, x, y \in X$

Lemma 2.4 Let X be a medial BCI-algebra X then $0 * (y * x) = x * y \quad \forall\, x, y \in X$

Definition 2.5 A non-empty subset I of a BCI-algebra X is said to be a BCI-ideal of X

if it satisfies $i)\quad 0 \in I \qquad ii)\quad x * y \in I$ and $y \in I$ implies $x \in I\ \forall\, x, y \in X$

Definition 2.6 A nonempty subset M of a medial BCI-algebra X is said to be a medial ideal of X if it satisfies $(i)\quad 0 \in M \quad ii)\ z * (y * x) \in M$ and $y * z \in M$ implies $x \in M \quad \forall\, x, y, z \in X$

Definition 2.7 Let X be a BCI-algebra. A fuzzy set σ in X is called doubt fuzzy BCI-ideal of X if it satisfies : $(i)\quad \sigma(0) \leq \sigma(x) \quad ii)\ \sigma(x) \leq \max\{\sigma(x * y), \sigma(y)\}$

$\forall\, x, y, z \in X$

III. MAIN RESULTS

Definition 3.1 A nonempty subset M of a medial BCI-algebra X is said to be a medial

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K- ideal of X if it satisfies $(i)\quad 0 \in M \quad (ii)\ z * \big(y * (x * u)\big) \in M$ and $y * z \in M$ implies $x * u \in M\ \forall\, x, y, z \in X$

Definition 3.2 Let X be a BCI-algebra. A fuzzy set σ in X is called doubt fuzzy medial K-ideal of X if it satisfies $(i)\quad \sigma(0) \leq \sigma(x)$

$(ii)\ \sigma(x * u) \leq \max\left\{\sigma\left(z * \big(y * (x * u)\big)\right), \sigma(y * z)\right\}\ \forall\, x, y, z \in X$

Definition 3.3 A doubt intuitionistic fuzzy set $A = (\sigma_A, \rho_A)$ in X is called doubt intuitionistic fuzzy medial K-ideal of X if it is satisfies the following inequalities

$(i)\quad \sigma_A(0) \leq \sigma_A(x) \quad , \qquad \rho_A(0) \geq \rho_A(x)$

$(ii)\quad \sigma_A(x * u) \leq \max\left\{\sigma_A\left(z * \big(y * (x * u)\big)\right), \sigma_A(y * z)\right\}$

$(iii)\quad \rho_A(x * u) \geq \min\left\{\rho_A\left(z * \big(y * (x * u)\big)\right), \rho_A(y * z)\right\} \quad \forall\, x, y, z, u \in X$

Example 3.4 Let X = {0, a, b, c} be a set with a binary operation $*$ define by the following table :

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Define

X	0	a	b	c
σ_A	0.2	0.3	0.6	0.7
ρ_A	0.8	0.7	0.3	0.3

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Easily we can prove that $A = (\sigma_A, \rho_A)$ is a doubt intuitionistic fuzzy medial K-ideal of X.

Lemma 3.5 Let $A = (\sigma_A, \rho_A)$ be doubt intuitionistic fuzzy medial K-ideal of X. If $x \leq y * z$ in X, then $\sigma_A(x) \leq \sigma_A(y * z)$, $\rho_A(x) \geq \rho_A(y * z)$ $\forall\, x, y, z \in X$.

Proof : Let $x, y, z \in X$ be such that $x \leq y * z$, then $x * (y * z) = 0$

$$\sigma_A(x * 0) \leq \max\left\{\sigma_A\left(z * \left(y * (x * 0)\right)\right), \sigma_A(y * z)\right\}$$

That is $\sigma_A(x) \leq \max\{\sigma_A(z * (y * x)), \sigma_A(y * z)\} = \max\{\sigma_A(x * (y * z)), \sigma_A(y * z)\}$

$= \max\{\sigma_A(0), \sigma_A(y * z)\} = \sigma_A(y * z)$

Also, $\rho_A(x * 0) \geq \min\left\{\rho_A\left(z * \left(y * (x * 0)\right)\right), \rho_A(y * z)\right\}$

That is $\rho_A(x) \geq \min\{\rho_A(z * (y * x)), \rho_A(y * z)\} = \min\{\rho_A(x * (y * z)), \rho_A(y * z)\}$

$= \min\{\rho_A(0), \rho_A(y * z)\} = \rho_A(y * z)$

This completes the proof.

Lemma 3.6 Let $A = (\sigma_A, \rho_A)$ be doubt intuitionistic fuzzy medial K-ideal of X, if the inequality $x * (y * z) \leq u$ hold in X, then $\sigma_A(x) \leq \max\{\sigma_A(u), \sigma_A(y * z)\}$,

$\rho_A(x) \geq \min\{\rho_A(u), \rho_A(y * z)\}$ $\forall\, x, y, z, u \in X$

Proof : Let $x, y, z, u \in X$ be such that $x * (y * z) \leq u$. Thus put u = 0 in condition (ii) ,

we get $\sigma_A(x * 0) \leq \max\left\{\sigma_A\left(z * \left(y * (x * 0)\right)\right), \sigma_A(y * z)\right\}$

That is $\sigma_A(x) \leq \max\{\sigma_A(z * (y * x)), \sigma_A(y * z)\} = \max\{\sigma_A(x * (y * z)), \sigma_A(y * z)\}$

$\leq \max\{\sigma_A(u), \sigma_A(y * z)\}$

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Also, $\rho_A(x*0) \geq \min\left\{\rho_A\left(z*\left(y*(x*0)\right)\right), \rho_A(y*z)\right\}$

That is $\rho_A(x) \geq \min\{\rho_A(z*(y*x)), \rho_A(y*z)\} = \min\{\rho_A(x*(y*z)), \rho_A(y*z)\}$

$\geq \min\{\rho_A(u), \rho_A(y*z)\}$

Theorem 3.7 Every doubt intuitionistic fuzzy medial K-ideal of X is doubt intuitionistic fuzzy subalgebra of X.

Proof : Let $A = (\sigma_A, \rho_A)$ be doubt intuitionistic fuzzy medial K-ideal of X. Since $x*y \leq x$ for all $x, y \in X$, then $\sigma_A(x*y) \leq \sigma_A(x)$ and $\rho_A(x*y) \geq \rho_A(x)$

Put u = 0 in condition (ii) , we get

$\sigma_A(x*y) \leq \sigma_A(x) \leq \max\left\{\sigma_A\left(z*\left(y*(x*0)\right)\right), \sigma_A(y*z)\right\}$

$= \max\{\sigma_A(z*(y*x)), \sigma_A(y*z)\}$

Again we put z = 0, $\sigma_A(x*y) \leq \max\{\sigma_A(0*(y*x)), \sigma_A(y*0)\}$

$= \max\{\sigma_A(x*y), \sigma_A(y)\} \leq \max\{\sigma_A(x), \sigma_A(y)\}$

Also, $\rho_A(x*y) \geq \rho_A(x) \geq \min\left\{\rho_A\left(z*\left(y*(x*0)\right)\right), \rho_A(y*z)\right\}$

$= \min\{\rho_A(z*(y*x)), \rho_A(y*z)\}$

Again we put z = 0, $\rho_A(x*y) \geq \min\{\rho_A(0*(y*x)), \rho_A(y*0)\}$

$= \min\{\rho_A(x*y), \rho_A(y)\} \geq \min\{\rho_A(x), \rho_A(y)\}$

Hence $A = (\sigma_A, \rho_A)$ is doubt intuitionistic fuzzy subalgebra of X.

Theorem 3.8 Let $A = (\sigma_A, \rho_A)$ be doubt intuitionistic fuzzy ideal of X, such that $\sigma_A(x) \leq \max\{\sigma_A(y), \sigma_A(z)\}$, $\rho_A(x) \geq \min\{\rho_A(y), \rho_A(z)\}$ and the inequality $x*y \leq z$

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are satisfied for all $x, y, z \in X$. Then $A = (\sigma_A, \rho_A)$ is doubt intuitionistic fuzzy medial

K-ideal of X.

Proof : Let $A = (\sigma_A, \rho_A)$ be doubt intuitionistic fuzzy ideal of X. Recall that

$\sigma_A(0) \leq \sigma_A(x)$ and $\rho_A(0) \geq \rho_A(x)$ for all $x \in X$.

Since, $(x * u) * \Big(z * \big(y * (x * u)\big)\Big) = \big(y * (x * u)\big) * \big(z * (x * u)\big) \leq y * z$, it follows from the hypothesis that $\sigma_A(x * u) \leq \max\Big\{\sigma_A\Big(z * \big(y * (x * u)\big)\Big), \sigma_A(y * z)\Big\}$

$$\rho_A(x * u) \geq \min\Big\{\rho_A\Big(z * \big(y * (x * u)\big)\Big), \rho_A(y * z)\Big\}$$

Hence $A = (\sigma_A, \rho_A)$ is doubt intuitionistic fuzzy medial K-ideal of X.

References

[1]. K.T.Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and systems, 1986; 20(10) : 87-96
[2]. T. Bej and M.Pal, Doubt intuitionistic fuzzy ideals in BCK/BCI – algebrasCommunicated.
[3]. K.Iseki, On BCI-algebras,Math. Sem. Notes Kobe Univ. 8 (1980) 125-130
[4]. K.Iseki and S. Tanaka, An Introduction to the theory of BCK – algebras, Math. Japon. 23(1)(1978/79) 1- 26
[5]. Samy M. Mostafa, Abdelaziz E.Radwan, Amany M.Menshawy, Reham Ghanem, 2015, "Doubt Intutionistic Fuzzy Magnified Translation Medial Ideals in BCI-algebras", J.Math . Comput. Sci.5, No.3, pp.297-319.
[6]. L.A.Zadeh , Fuzzy sets, Inform. Control 8 (1965) 338-353