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Adaptive Fractional Fourier Filter

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Abstract. The fractional Fourier transform (FRFT) is a linear transformation generalizing the Fourier transform. In this paper an algorithm has been purposed to perform the filtering of time-varying signals using Fractional Fourier Transform (FRFT). Mean square error (MSE) and cross-correlation are the parameters that used to match the recovered signals. And the concept of adaptively choose the cut-of-frequencies for the band pass filter in every Fractional Fourier Domain (FRFD) has been introduced, depending upon the desired frequencies. The proposed filtering algorithm able to reduces the MSE by a factor of 2 for a single having one frequency component, and reduces the MSE by a factor of 21 for a signal having five frequency components. The normalized value of cross-correlation is also increased by a factor of 0.6601 for single having one frequency component, and with a factor of 0.7247 for a signal having five frequency components. These results have been found while comparing the recovered signal from FIR filter of same order.

Keywords: Fractional Fourier transform, fractional Fourier domain, adaptive fractional Fourier filter (AFFF), filtering.

1 Introduction

After classical Fourier Transform (FT) get generalized it results into the FRFT. The concept of generalization of FT was given by V. Namias in [1]. The one important parameter of FRFT is ‘α’ variation in this parameter allow the spectrum rotation of signal in the time-frequency plane. The FT is one of the important tools for the analysis of signals [2]. To analyze the signal having time-varying property, researchers commonly uses the time-frequency plane, in which time axes and frequency axes are orthogonal to each other [3]. Two successive operations of forward FT provide the original signal in its reflected version, the FT rotate the signal by right angle in time–frequency plane and shows the orthogonal representation of the sinusoidal signal. Where, FRFT rotate the signal by any angle in continuous time–frequency plane, and shows the orthonormal representation of the chirp signal. The rotational Fourier transform is another name of FRFT. Angular Fourier transform also refer to FRFT in the present state-of-the-art articles. FRFT and other time-varying signal analysis tools, such as Wigner distribution [3], short-time Fourier transform, wavelet transform [4], are belong to same category. FRFT has many applications, to solve the differential equations [4], in the field of quantum mechanics [1], to process the optical signal [5], in swept-frequency filters [6], for pattern recognition in image processing [7], to analyze the signal in time–frequency feild [8]–[10], improvement of pervasive mobile robots [13]. A summary of FRFT properties in the signal analysis given in [6] and an analysis of signal in fractional Fourier domain is discussed in [12].

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In this paper, section II is about the basic concept of FRFT, in section III the proposed filtering algorithm has been discussed with the block diagram and also with the step form of the algorithm. And section VI is about the simulations and results where the filtering results for a signal having one frequency component and a signal having three frequency components have been discussed, also this section about the analysis of the results on the bases of statistical discussion of the maximum value of MSE and the statistical parameter cross-correlation for some signal. In section V, include the conclusion and future work.

2Basic Concept of FRFT

The FRFT with angle parameter α of a signal $f(t)$ is defined in (1):

$$F^{\alpha}(u)=\begin{cases}\sqrt{\frac{1-jcot\,(\alpha)}{2\pi}}\int_{-\infty}^{\infty}f(t)e^{\frac{j(t^2+u^2)\cot(\alpha)-2tucsc\,(\alpha))}{2}}dt\,,if\alpha\neq k\pi\\ f(u) \qquad ,if\ \alpha=2k\pi\\ f(-u) \qquad ,if\ \alpha=(2k+1)\pi\end{cases} \qquad (1)$$

Where $F^{\alpha}(u)$is FRFT of $f(t)$ with order α,and $\alpha = a\pi/2$, where 'a' is the order of FRFT. The range of 'a' is [-2, 2].

Few special cases of FRFT, as follow:

Null rotationat $a=0$ or 4; α become 0 or 2π; then $F^{\alpha}(u)$will act asan identity operator. Due to which$F^{0}(u)$ is exactly same as $f(t)$. Where $f(t)$ and$F^{0}(u)$ is known as the FRFT pair at $\alpha=0$.

FT using FRFT at $a = 1$;α become $\pi/2$; then $F^{\alpha}(u)$will act asa classic Fourier operator. Where $F(j\omega)$and $f(t)$is known as the Fourier Transform pairor FRFT pair at$\alpha=\pi/2$.

Flipped operation/ time inversion at $a=2$;α become π; then $F^{\alpha}(u)$will do the time inversion operation on $f(t)$. Which result in $f(-t)$. Where $f(-t)$ is the flipped version of $f(t)$.$F^{\pi}(u)$ and $f(t)$is known as the Fourier Transform pairor FRFT pair at $\alpha=\pi$.

Inverse Fourier Domain at $a=3$;become $3\pi/2$; then $F^{\alpha}(u)$will act asa classic inverse Fourier operator. Where $F(-j\omega)$and $f(t)$is known as the FRFT pair at $\alpha=3\pi/2$..

3Proposed Filtering Algorithm

As the FRFT is used to perform the spectrum rotation of the signal in time-frequency plane, this property of the FRFT is used to rotate the noisy signal at different FRFD and apply the band pass FIR filter on signal in every FRFD. This concept of filtering in FRFD is given in [2].

The proposed filtering algorithm is called as Adaptive Fractional Fourier Filter (AFFF or A3F) or Ajmer's Fractional Fourier Filter (AFFF or A3F). The prefix adaptive is used because the filter coefficients are changed adaptively for different FRFDs.

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The proposed filtering algorithm is based on two assumptions. First assumption is that the frequency of noise signal should be different from the original signal at every instant of time. This assumption is practically possible, because if the noise signals of same frequency components are added to the signal then this type of noise does not affect the original signal. The second assumption is that the high frequency component and the low frequency component of the desired signal are known. This assumption is also practically possible because this is always knowing the frequency band for the particular application of the signals (e.g. for speech signal high frequency component is 3.3 KHz and low frequency component is 330Hz).

a) Description of AFFF/A3F – Algorithm

The A3F mainly consist of two parts one is a digital filter to filter the received signal and second is the adaptive algorithm to regenerate the filter coefficients of the digital filter. The proposed A3F – algorithm is different from the existing adaptive algorithms (like LMS, RMS etc.).

The basic block diagram for A3F is illustrated in Figure 1. Where $d(n)$ is the desired signal, $x(n)$ is the original/input signal, $y(n)$ is the output signal, $X^{\alpha}(u)$ is the FRFT of $x(n)$ with parameter α, $Y^{\alpha}(u)$ is the output of the digital filter, $y(n)'$ is the Inverse Fractional Fourier Transform (IFRFT) of $Y^{\alpha}(u)$ with same value of α, C is the vector for filter coefficients, $x(n)'$ is equal to the $y(n)'$ and known as the input to delay function instead of $x(n)$ for next iteration. The value of $\alpha/$'a' is controlled by adaptive algorithm.

In Figure 1 the block named as Delay functions, FRFT, Digital filter, and IFRFT are well known. But the block named as Adaptive Algorithm is new. Now about it, the primary input to this block is $d(n)$. On the basis of $d(n)$ it finds the filter coefficients. After that the 'a' value is compared with 1, if it's equal to 1 then the $y(n)'$ is given as the output of the system, and if not equal to 1 then the 'a' ('a' lies between 0 and 1) value is increment by some fraction value. The $y(n)'$ is given back to delay function as $x(n)'$. The internal block diagram of adaptive algorithm is illustrated in Figure 5.

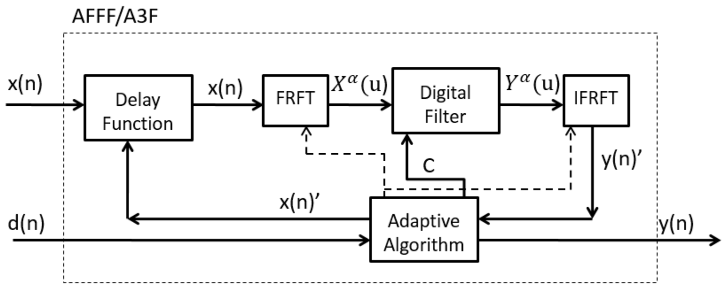

Figure 1: Basic block diagram of AFFF/A3F algorithm.

In Figure 2 $D^{\alpha}(u)$ is the FRFT of $d(n)$ with parameter α. The adaptive algorithm block first take the FRFT of the desired signal $d(n)$, then find the lower frequency (FL) and higher frequency (FH) components in$D^{\alpha}(u)$. Using these FL and FH it finds the higher cut-of-frequency (FH_c) and lower cut-of-frequency

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(FL_c) for designing a FIR band pass filter, now the filter coefficients C are passed to the digital filter block in Figure 1. When the output of digital filter $Y^{\alpha}(u)$ comes and taken its IFRFT then the output is $y(n)'$ that goes to adaptive algorithm block. In Figure 2 the block 'round toward negative infinity' is used to round the value of 'a' toward negative infinity (e.g. If $a = 0.3$ the output is 0, if $a = 0.7$ the output is 0 and if $a = 1.1$ then the output is 1). After that the AND gate is used to compare the value of 'a' with 1, if $a \neq 1$ then switch S1 is on and the $y(n)'$ become $x(n)'$ for next iteration. But if $a = 1$ then switch S2 is on and it gives the output of AFFF as $y(n) = y(n)'$.

b) Step Form of AFFF/A3F - Algorithm

The last section is a discussion about the A3F – algorithm. In this section the summarized step form of the A3F – algorithm is as follow:

Step 1: Take the FRFT of the desired signal $d(n)$.

$$D^{\alpha}(u) = F^{\alpha}[d(n)]$$

Where $F^{\alpha}[\,]$ is the FRFT operator at α, and $\alpha = \frac{a\pi}{2}$.

Step 2: Find the lower frequency (FL) and higher frequency (FH) components in$D^{\alpha}(u)$. Using FL and FH find the lower cut-of-frequency (FL_c) and higher cut-of-frequency (FH_c).

Step 3: Find the digital filter coefficients (C) using these FL_c and FH_c.

Step 4: Find the FRFT of input signal $x(n)$.

$$X^{\alpha}(u) = F^{\alpha}[x(n)]$$

Step 5: Filter the $X^{\alpha}(u)$ using the filter coefficients C. Which give the$Y^{\alpha}(u)$.

Step 6: Take the IFRFT of$Y^{\alpha}(u)$.

$$y(n)' = F^{(2\pi-\alpha)}[\,Y^{\alpha}(u)]$$

Step 7: If $a = 1$, go to step 11.

Step 8: If $a < 1$, increment 'a' by small value Δa

$$a = a + \Delta a$$

Step 9: Change the input signal $x(n)'$ as $y(n)'$ for next iteration.

$$x(n)' = y(n)'$$

Step 10: Go to step 1.

Step 11: $y(n)$ is the final output of AFFF/A3F.

$$y(n) = y(n)'$$

These eleven steps summarized the AFFF/A3F – algorithm. Using A3F the filtering of noise from time-varying signals become more efficient as compared to the simple filters.

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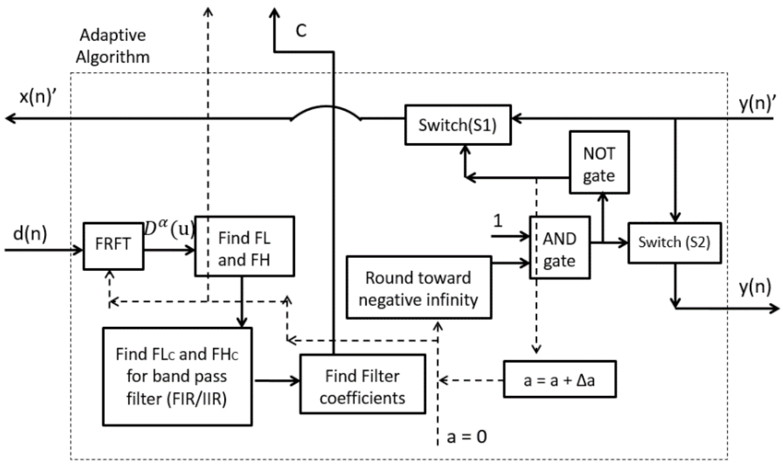

Figure 2: Internal block diagram of the adaptive algorithm.

4 Simulations and Results

In this section, the simulations results are presented and discussion on the results. The testing is done by taking two signals; one is which has single frequency component and second which have three frequency components. The output given by AFFF is compared with the output of FIR band pass filter.

a) For One Frequency Component

In this test the signal taken as a sine wave of a single frequency and the noise taken as chirp signal. In Figure 3(a) the spectrogram of a sine wave, Figure 3(b) the spectrogram of a chirp signal and then the noise as chirp signal added to the sine wave the resulting noisy signal spectrogram shown in Figure 3(c). First the filtering is done by using the FIR band pass filter the resulting spectrogram shown in Figure 3(d).

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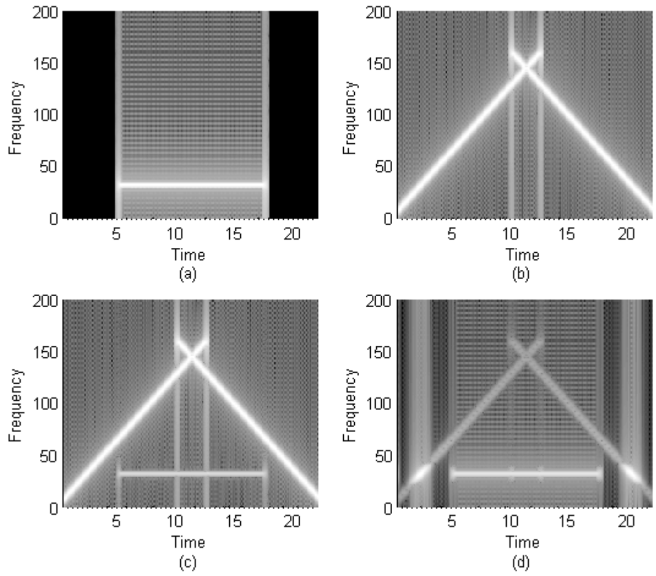

Figure 3: (a) the spectrogram of a sine wave, (b) the spectrogram of the chirp signal, (c) The required noisy signal and (d) Filtering of the noisy signal using FIR band pass.

In Figure 3(d) the most of the noise get removed by pass band FIR filter, where the remaining noise can be removed using the AFFF – algorithm. In Figure 4 the six results show for the six different values of 'a'. The filtered signal in Figure 3(d) is input for the next iteration of the AFFF.

In Figure 4 the six results for increasing value of 'a' are given. The filtering result of AFFF at a = 0 in Figure 4(a) is same as the filtering result of FIR band-pass filter is shown in Figure 3(d). After that from Figure 4(b) to Figure 4(f), there was a small portion of noise, which was removed in next iteration as compared to the last one. Now from Figure 4(f) it is clear that the original signal that shown in Figure 3(a) is recovered. We are just visualizing the results for the AFFF the statistical discussion will be given in next section.

b) For Three Frequency Components

This section, shows how the AFFF give the good results for one frequency components signal. So, the signal taken as a sine wave of three frequencies and the noise taken as the same chirp signal. In Figure 5(a) the spectrogram of a sine wave, Figure 5(b) the spectrogram of a chirp signal and then the noise as chirp signal added to the sine wave the resulting noisy signal spectrogram shown in Figure 5(c). First the filtering is done by using the FIR band pass filter the resulting spectrogram shown in Figure 5(d).

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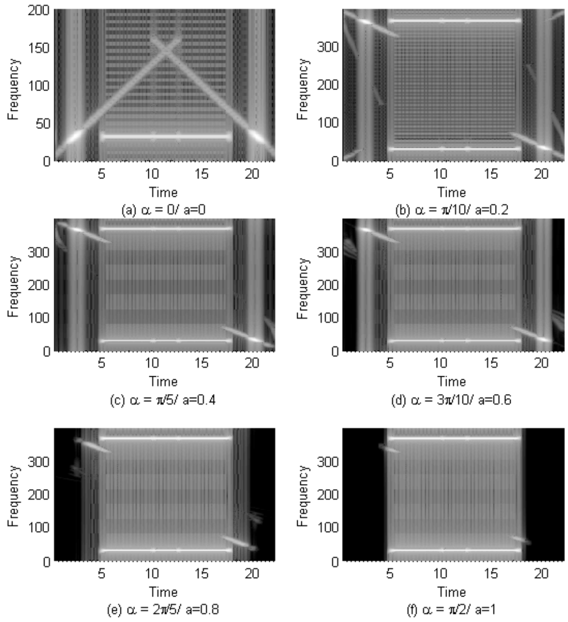

Figure 4: Six different FRFD results after filtering using AFFF

In Figure 5(d) the most of the noise removed by FIR band pass filter, but as compared to the results for filtering of one frequency signal there is more noise, which shows that if the signal has more than one frequency components then the noise removed by the FIR band pass filter in time domain give higher MSE value, that explained in next section. The Figure 6 shows that the noise that cannot be removed using FIR band pass filter in the timedomain can be removed easily using the proposed AFFF – algorithm.

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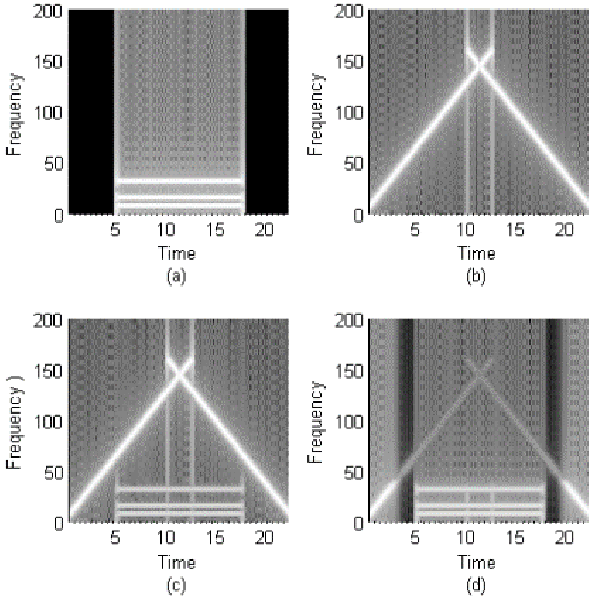

Figure 5: (a) the spectrogram of a sine wave having three frequencies, (b) the spectrogram of the chirp signal, (c) The required noisy signal and (d) Filtering of the noisy signal using FIR band pass.

In Figure 6 the six results for increasing value of 'a' are given. The filtering result of AFFF at a = 0 in Figure 6(a) is same as the filtering result of FIR band-pass filter shown in Figure 5(d). After that from Figure 6(b) to Figure 6(f), there is a small portion of noise is removed in next iteration as compared to the last one. Now from Figure 6(f) if is clear that the original signal that shown in Figure 5(a) is recovered. We are just visualizing the results for the AFFF the statistical discussion will be given in next section.

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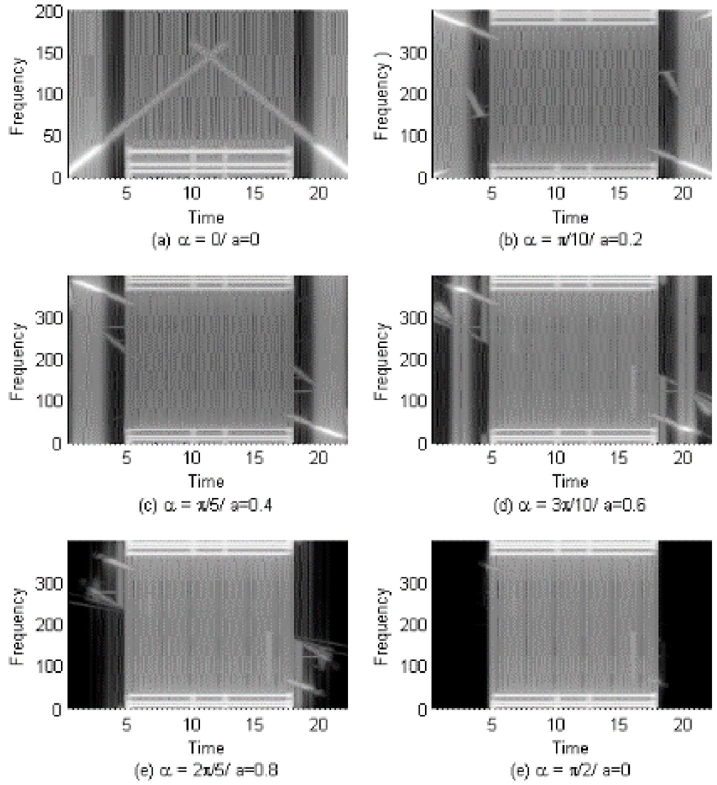

Figure 6: Six different FRFD results after filtering using AFFF

From the above discussion, it is clear that for a signal having high-frequency band can be filtered precisely using AFFF rather than using time domain filter. In Figure 7 the graph for statistical data given in Table 1.

TABLE I. COMPARISON OF MAX(MSE) OF AFFF WITH FIR BAND PASS FILTER

Type of Filter	For signal having One frequency Max(MSE)	For signal having Three frequencies Max(MSE)	For signal having five frequencies Max(MSE)
FIR Band Pass	2.1221	10.2748	21.8275
AFFF at a=0	2.1221	10.2748	21.8275
AFFF at a=0.1	0.0195	6.3967	2.7857
AFFF at a=0.2	0.0001	1.0793	3.7781
AFFF at a=0.3	0.0001	0.0016	0.5404
AFFF at a=0.4	0.0002	0.0008	0.8058

AFFF at a=0.5	0.0002	0.0009	0.0780
AFFF at a=0.6	0.0002	0.0009	0.0074
AFFF at a=0.7	0.0002	0.0009	0.0014
AFFF at a=0.8	0.0001	0.0007	0.0016
AFFF at a=0.9	0.0001	0.0005	0.0012
AFFF at a=1	0.0000	0.0002	0.0004

Now, the MSE is one of the statistical parameters that tells about the error present in the recovered signal as compared to the original signal. As smaller is the MSE as better is the method of the filter which is used to recovered the signal. But there is another statistical parameter that tells how the recovered signal is matched with the original signal known as cross-correlation. In another way, we can say that the cross-correlation of two signals tells how much they correlated to each other. So if the cross-correlation of recovered signal with the original signal is higher, then the method/ algorithm that used to recover the signal are good.

So, in Table 2 the Cross-correlation of recovered signal with the original signal for FIR band pass and for all the iteration of AFFF is given. As the number of iterations increases the cross-correlation value increase that means using AFFF we can recover the signal which cannot be recovered using time domain filters.

The similar type of analysis and testing is also done for the signal having five frequencies. The statistical results for a signal having five frequencies are also discussed in next section.

a) The Statistically Discussion

In previous sections, the graphical representation had been explained. Now in this section with the help of statistical discussion, it will be prove that the proposed AFFF is better than the time domains filter. First of all the comparison between the Max(MSE) that is attained after every iteration of AFFF to the Max(MSE) that attained after filtering with time domain filter is shown in Table 1.

The results are given in Table 1 at proving that the results of AFFF are better than the simple time domain filters. As the second row of the Table 1having the Max(MSE) value when to filter the noisy signal with the time domain signal, and the last row having the values for the Max(MSE) when to filter the noisy signal using AFFF after 10th iteration. So the Max(MSE) is reduced by a factor of 2 for the signal having one frequency, and with a factor of 10 & 21 for the signal having three & five frequencies components respectively. As we increasing the number of frequencies in the signal that means the frequency band is increasing then the AFFF give more precise results as compared to the time domain filter.

TABLE II. MAX CORRELATION OF THE RECOVERED SIGNAL USING AFFF AND FIR BAND PASS FILTER

Type of Filter	For signal having One frequency	For signal having Three frequencies	For signal having five frequencies
FIR Band Pass	0.3396	0.2582	0.2377

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AFFF at a=0	0.3396	0.2582	0.2377
AFFF at a=0.1	0.7160	0.2788	0.2387
AFFF at a=0.2	0.7904	0.3456	0.2406
AFFF at a=0.3	0.7899	0.3843	0.2467
AFFF at a=0.4	0.7894	0.4681	0.3191
AFFF at a=0.5	0.7890	0.6223	0.3511
AFFF at a=0.6	0.8221	0.9311	0.3849
AFFF at a=0.7	0.9986	0.9993	0.4296
AFFF at a=0.8	0.9996	0.9994	0.4996
AFFF at a=0.9	0.9996	0.9994	0.6353
AFFF at a=1	0.9997	0.9994	0.9624

From the Table 2, it is clear that the cross-correlation is increased by a factor of 0.6601, 0.7412 and 0.7247 for a signal having one, three and five frequency components respectively.

5 Conclusion

The conclusion of the work is that the proposed filtering algorithm AFFF/A3F is able to give the better results as compared to the time domain filters. And can able to remove that noise which cannot be removed in the time domain, because the FRFT helps to rotate the spectrum of time domain signal. The simulation results for three different signal having different frequency components or different frequency band are filtered using AFFF that gives the improved results as compared to the FIR band pass filter. In results, the MSE reduced by a factor of 21 for a signal having five frequency components. And the cross-correlation is improved / increases by a factor of 0.7247 for the same signal.

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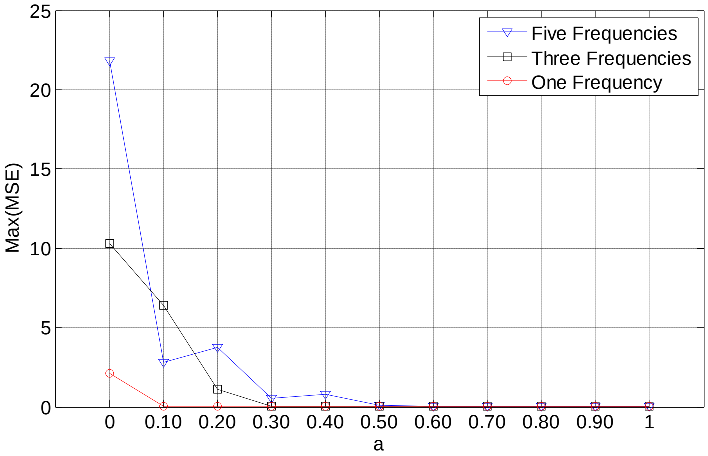

Figure 7: Max(MSE) for three different signals having one, three and five frequencies.

By using proposed filter(AFFF/A3F) one can enhance any practical signal like ECG signal, Radio signal, Speech signal etc. Because the AFFF is work for that type of signal that has a group of frequency or having as a specific frequency band. For example, one can apply the AFFF – algorithm on a noisy speech, the speech signal always has the frequency band of 330Hz to 3.3KHz. Similarly, speech signals one can apply AFFF – algorithm on any type of practical signal whose frequency band is known.

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