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A Chaotic Study on One-Dimensional Heisenberg Ferromagnetic Spin Chain

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ABSTRACT

The Chaotic behavior of a Heisenberg anisotropic ferromagnetic spin chain with bilinear interaction is investigated in detail. A model Hamiltonian is proposed which was then bosonized using Holstein-Primakoff transformation. Remodeling the Hamiltonian into first quantized form assist to study the Chaotic behavior of the system. The studies are then carried out by plotting the Phase diagrams and Chaotic trajectories. Maximal Lyapunov Exponent analysis is employed to analyze the stability for various perturbation size.

Keywords: Heisenberg Ferromagnetic, Lyapunov

1. INTRODUCTION

During the past few decades, tremendous progress has been made in identifying nonlinear spin excitations in one-dimensional ferromagnetic systems [1-3]. In a ferromagnetic system the spins of all atoms in ground state are oriented in one direction so one wants naturally to understand the behaviour of the excited states. Much effort has been put forth in understanding the nonlinear mechanisms and rich variety of spin excitations in different spin systems. Chaos and Nonlinear Dynamics have provided new theoretical and conceptual tools that allows us to capture, understand, and link together the surprisingly complex behaviours of simple systems namely chaos in essentially every field of contemporary science. It is the phenomenon of occurence of bounded non periodic evolution in completely deterministic nonlinear dynamical systems with high sensitive dependence on initial conditions. Some areas that are benefiting from chaos are computer science, economics, engineering, finance, meterology, philosophy, biology etc [4]. However the evolution equations governing the dynamics of the ferromagnetic systems are in general highly nonlinear differential equations. The manuscript is framed with the detailed investigation on One-dimensional Heisenberg Ferromagnetic spin chain with bilinear interaction.

2. An analysis on Heisenberg Ferromagnetic Spin Chain with bilinear interaction

The model Hamiltonian to describe the interpretation on Heisenberg Ferromagnetic Spin Chain with anisotropic and bilinear interaction can be written as

$$H = -\sum_i [\tilde{J}(\vec{S}_i.\vec{S}_{i+1}) - \tilde{A}(S_i^z)^2] \qquad (1)$$

The spin operator at the lattice site *i* can be represented by $\vec{S}_i = (S_i^x, S_i^y, S_i^z)$, *J* is the constant coefficient of bilinear exchange interaction and A is the anisotropic parameter that corresponds to the uniaxial crystal field anisotropy. The exchange interaction which is highly responsible for the parallel alignment of spins in a ferromagnetic system is the bilinear exchange interaction. To express the spin Hamiltonian (1) in the dimensionless form the dimensionless spin $\hat{S}_i = \frac{\vec{S}_i}{\hbar}$ and the spin raising and lowering operators $\hat{S}^{\pm} = \hat{S}^x \pm i\hat{S}^y$ are instigated. The resulting Hamiltonian is given by

$$H = -\sum_i \left[\frac{J}{2S^2}(\hat{S}_i^+\hat{S}_{i+1}^- + \hat{S}_i^-\hat{S}_{i+1}^+ + 2\hat{S}_i^z \hat{S}_{i+1}^z) - \frac{A}{S^2}(\hat{S}_i^z)^2\right]. \quad (2)$$

For the bosonic transformation of spin operators, Holstein-Primakoff representation [5] is executed. The following relations bosonise the Hamiltonian.

$$\begin{aligned}\hat{S}_n^+ &= \sqrt{2}S(1 - \frac{\epsilon^2}{4}a_n^\dagger a_n)\epsilon a_n \\ \hat{S}_n^- &= \sqrt{2}S\epsilon a_n^\dagger(1 - \frac{\epsilon^2}{4}a_n^\dagger a_n), \\ \hat{S}_n^z &= S - a_n^\dagger a_n.\end{aligned} \quad (3)$$

After substitution the Hamiltonian is now in the second quantised form. This recasted equation needs further expansion to study the chaotic dynamics of the system.

$$\begin{aligned}a_n^\dagger &= \left(\frac{M\omega}{2\hbar}\right)^{\frac{1}{2}} x - i\left(\frac{1}{2M\hbar\omega}\right)^{\frac{1}{2}} p_x, \\ a_n &= \left(\frac{M\omega}{2\hbar}\right)^{\frac{1}{2}} x + i\left(\frac{1}{2M\hbar\omega}\right)^{\frac{1}{2}} p_x, \\ a_{n+1}^\dagger &= \left(\frac{M\omega}{2\hbar}\right)^{\frac{1}{2}} (x+h) - i\left(\frac{1}{2M\hbar\omega}\right)^{\frac{1}{2}} p_x, \\ a_{n+1} &= \left(\frac{M\omega}{2\hbar}\right)^{\frac{1}{2}} (x+h) + i\left(\frac{1}{2M\hbar\omega}\right)^{\frac{1}{2}} p_x.\end{aligned} \quad (4)$$

To study the theory of excitations in the magnetic systems Hamilton's equations of motion plays a major role. They are

$$\begin{aligned}\frac{dx}{dt} &= \frac{\partial H}{\partial p_x}, \\ \frac{dp_x}{dt} &= -\frac{\partial H}{\partial x}\end{aligned} \quad (5)$$

A set of singular points are then calculated from the obtained equations of motion for the further plotting of phase plots and trajectories.

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Table 1: Singular points

Sl.no	x	px
(i)	-1.56588	0
(ii)	-0.0386759	0
(iii)	0.000276243	0
(iv)	0.0394837	0
(v)	1.56354	0

Phase plots:

For the variation in anisotropic interaction parameter (A), the following analysis is characterized. By applying two different conditions, that is when A=0.09 and A=0.1 the phase portraits and the Lyapunov stability curves are drawn.

Case 1: A=0.09

Figs. 1(a) and 1(b) are the Phase plots obtained when A=0.09. The unperturbed plot is a closed type elliptical orbit. It commences periodic motion with same frequency. For the perturbed study the values x = 0.000276243 and p_x =0 are chosen. When perturbation is applied the elliptical orbit turns into rhombus shaped curve with smooth edges. One edge touches Zero and the other side adjoins the displacement value x = 13.

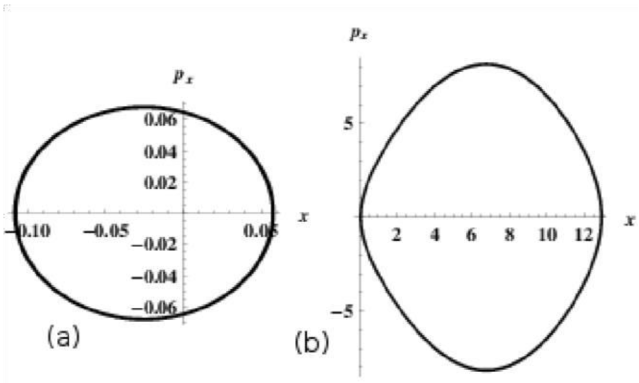

Figure 1: Phase Space Plots (a) unperturbed (b) perturbed for $A = 0.09, J = 12, S = 0.5, \varepsilon = 0.5, x = 0.000276243, p_x = 0$

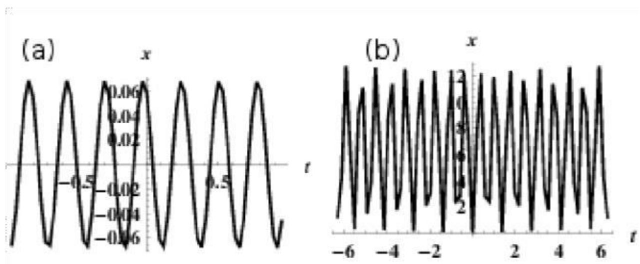

Figure 2: Trajectory Plot (a) unperturbed (b) perturbed for $A = 0.09, J = 12, S = 0.5, \varepsilon = 0.5, x = 0.000276243, p_x = 0$

The phase space trajectories obtained for this system are plotted in Figs. 2(a) and 2(b). Particularly the unperturbed plot exhibits a periodic motion of waves with similar amplitude and frequency. When perturbed as in Fig. 2(b) the trajectory shows quasi periodic property.

Case 2: A=0.1

To find out whether anisotropic interaction plays a role in the chaotic property of this system we now alter the value of A parameter as 0.1. Figs. 3 and 4 are the phase plots and phase trajectories attained. The unperturbed plot for A = 0.1 has no significant change when compared with 0.09. Both the curves reveal linear behaviour. Fig. 3(b) is the perturbed curve with one edge touching its axis. The other point reaches at x = 11. Both the trajectories observed are with same frequency and amplitude. The unperturbed plot has non sticky trajectories and the perturbed plot has sticky trajectories. They behave periodically.

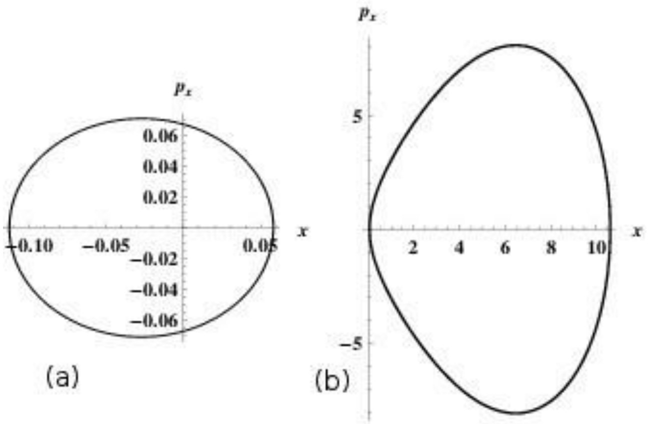

Figure 3: Phase Space Plots (a) unperturbed (b) perturbed for $A = 0.1, J = 12, S = 0.5, \varepsilon = 0.5, x = 0.000276243, p_x = 0$

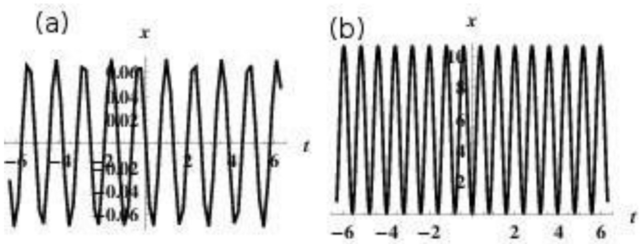

Figure 4: Trajectory Plot (a) unperturbed (b) perturbed for $A = 0.1, J = 12, S = 0.5$ $, \varepsilon = 0.5, x = 0.000276243, p_x = 0$

Lyapunov Stability Analysis:

Lyapunov Stability analysis plays a major role to study the complexity and stability of dynamical systems. Figs. 5 and 6 are the Lyapunov curves in terms of displacement perturbation with different size of perturbation. While varying its size it is found that there are changes in the respective trajectories. The effect of perturbation is studied when the displacement size varies for 10−11,10−13 and 10−15 at A = 0.09 and 10−10,10−13 and 10−18 at A = 0.1. Next the impact of adding perturbation in momentum and its effect on the stability of the system is analyzed. Figs. 7 and 8 shows the LCE curves for momentum perturbation. When A = 0.09 as the size varies, the curve changes from increasing to decreasing state. The distorted Lyapunov spectra indicates chaoticity in the system.

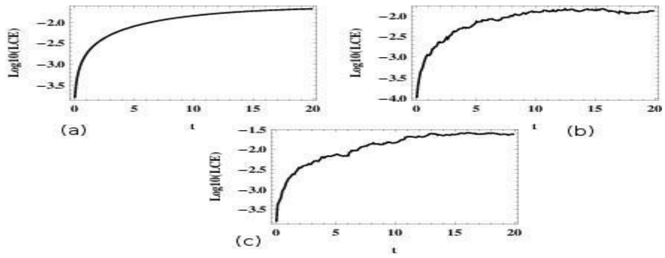

Figure 5: Lyapunov curves interms of displacement perturbation for A=0.09,J = 12,S = 0.5, ε = 0.5 at (a) 10^{-11} (b) 10^{-13} (c) 10^{-15}

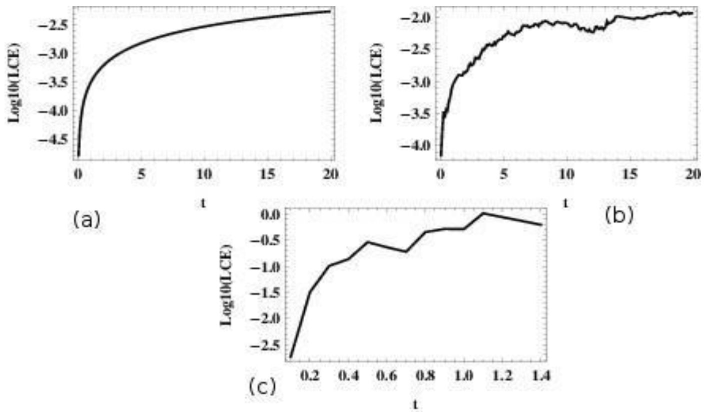

Figure 6: Lyapunov curves interms of displacement perturbation for A = 0.1,J = 12,S = 0.5, ε =0.5 at (a) 10^{-10} (b) 10^{-13} (c) 10^{-18}

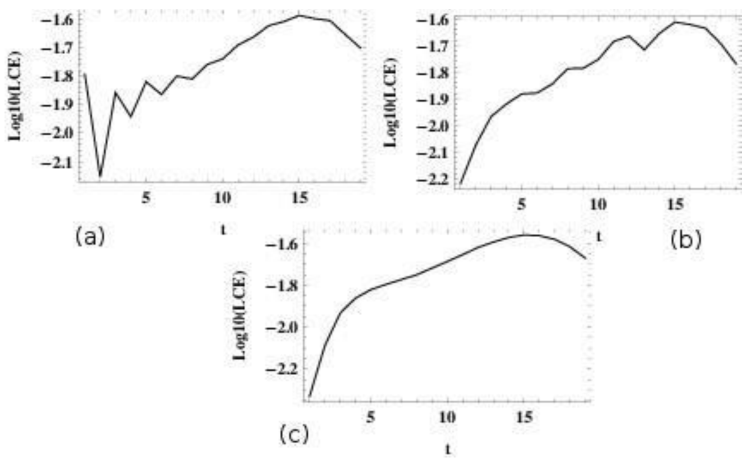

Figure 7: Lyapunov curves interms of momentum perturbation for A = 0.09,J = 12,S = 0.5, ε = 0.5 at (a) 10^{-11} (b) 10^{-13} (c) 10^{-15}

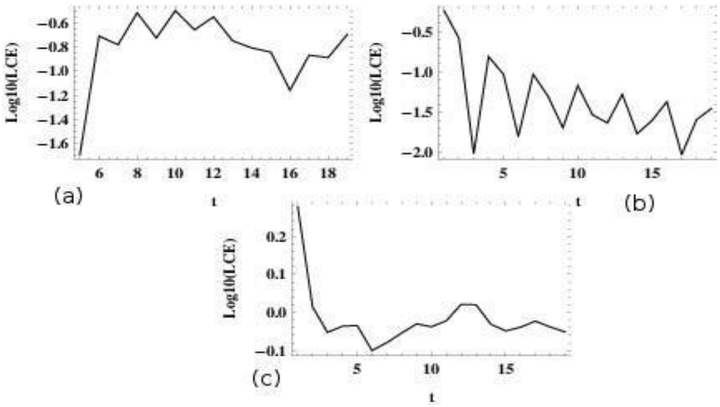

Figure 8: Lyapunov curves interms of momentum perturbation for A=0.1,J=12,S =0.5, ε = 0.5 at (a) 10^{-11} (b) 10^{-13} (c) 10^{-15}

3. CONCLUSION

This manuscript discuss briefly about the nonlinear spin dynamics of one dimensional Heisenberg FM spin chain with anisotropic and bilinear interactions at semi classical limit. From this investigation it is found that as the range of anisotropic interaction parameter varies the Phase plot exhibits periodic motion of waves with similar amplitude and frequency. The trajectories show quasi periodic property and from the LCE analysis the distorted Lyapunov Spectra indicates causticity in the system.

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