

# Slip Effects on Peristaltic Flow of a Williamson Fluid in a Channel Under the Effect of a Magnetic Field

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## INTRODUCTION

The study of peristaltic pumping has grabbed the notice of many researchers ever since the primary and foremost analysis of Latham [50]. A theoretical investigation truly for the peristaltic motion was carried out by Burns and Parkes [18] have modeled the pipe and a channeled flow of a Newtonian Fluid with peristalsis. A contemporary investigation was reported by Shapiro [64] for two - dimensional peristaltic pumping under conditions that the low Reynolds number that the flow may be considered as inertia free and the length of the have analysed the Williamson fluid flow with peristalsis in an asymmetric channel.

In recent years, the MHD motion of a liquid in a channel by way of resilient, regularly contracting walls is of significance in association with some troubles of the activity of physiological fluids. Subba Reddy and Gangadhar [80] have deliberated the symmetric channeled, Carreau fluid flow in an inclined in occurrence of a magnetic field with peristalsis. Recently, Subba Reddy et al. [81] have investigated the MHD effects on peristaltic pumping of Williamson fluid in a horizontal channel.

In many applications, the flow pattern corresponds to a slip flow the fluid shows a slaughter of sticking together at the wetted wall building the fluid slip along the wall. When the molecular average free path length of the fluid is comparable to the distance between the plates as in nano channels or micro channels, the fluid shows non-continuum effects such as slip-flow was verified practically by Derek et al. [20]. Newtonian fluid flow through a 2D micro channel with slip effects and peristalsis has been represented by Kwang [49]. Effect of slip on Maxwell fluid in a channel with peristalsis was considered by El sehaway et al. [31]. Slip Effects on the movement of pseudoplastic fluid with peristalsis and induced magnetic field in a channel were analyzed by Noreen et al [57].

Keeping all this in the view, we analysed the effects of slip and magnetic field on the Williamson fluid flow in a symmetric channel with peristalsis and lubrication approach. The flow is formulated in a wave frame of reference which also moves with swiftness of the wave. The expressions for the

The flow is unsteady in the laboratory frame  $(X, Y)$ . However, in a co-ordinate system moving

peristaltic wave is very long when compare with the width of a tube. An asymptotic solution for the peristaltic transport in a tube in terms of small wave number was presented by Yin and Fung [92] for a finite range of Reynolds number and wavelength. The small amplitude assumption has been relaxed under infinite wavelength approximation by Shapiro et al. [65]. Complete review on peristaltic pumping has been given by Jaffrin and Shapiro [47]. The lubrication-theory model is applicable globally in the limit of totally occluding peristaltic waves and is found velocity field and pressure gradient are obtained explicitly by applying perturbation series in terms of small Weissenberg number  $(We < 1)$ . The effects of Weissenberg number  $We$ , slip parameter  $\beta$ , Hartmann number  $M$  and amplitude ratio  $\phi$  on the pressure gradient and pumping characteristics are discussed with the aid of 2D plots.

## MATHEMATICAL OUTLINE OF THE PROBLEM

Let us think of the movement of Williamson fluid with peristalsis in a two-dimensional channel of width  $2a$  which is symmetric. The flow is caused by sinusoidal wave chain transmit with constant speed  $C$  along the channel walls. A consistent magnetic field  $B_0$  is taken in the direction perpendicular to the fluid flow. Small electrical conducting fluid is taken, so to facilitate the Reynolds number is small and the induced magnetic field can be ignored in assessment with the input magnetic field. Fig. 6.1 shows the graphical representation of channel.

The wall expression is as follows

$$Y = \pm H(X, t) = \pm a \pm b \cos \frac{2\pi}{\lambda}(X - ct) \quad (6.2.1)$$

Where  $b$  is the amplitude of the wave,  $\lambda$  - the wave length and  $X$  and  $Y$  - the rectangular coordinates with  $X$  measured along the axis of the channel and  $Y$  perpendicular to  $X$ . Let  $(U, V)$  be the velocity components in fixed frame of reference  $(X, Y)$ .

with the propagation velocity  $c$  (wave frame  $(x, y)$ ), the boundary shape is stationary. The relations which are useful in converting fixed frame to wave frame are as follows  $x = X - ct, y = Y, u = U - c, v = V$  (6.2.2)

Where  $(u, v)$  and  $(U, V)$  are velocity components in the wave and laboratory frames respectively.

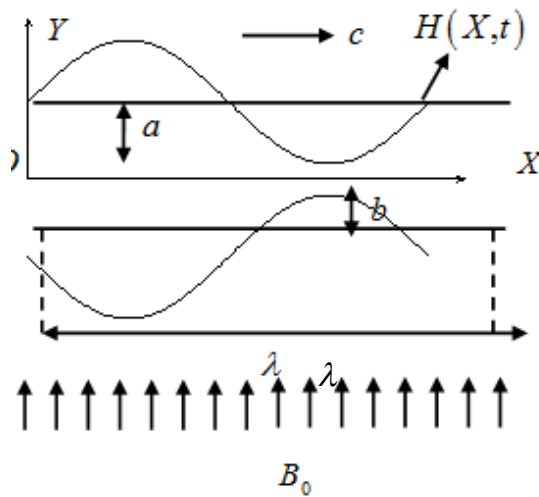


Fig. 6.1. The Physical Model

The Williamson fluid and its constitutive equation is

$$\tau = -[\eta_\infty + (\eta_0 + \eta_\infty)(1 - \Gamma \dot{\gamma})^{-1}] \dot{\gamma} \quad (6.2.3)$$

Where

$\tau$  - The extra stress tensor

$$\tau_{xx} = -2[1 + We \dot{\gamma}] \frac{\partial u}{\partial x}, \quad ,$$

$$\tau_{xy} = -[1 + We \dot{\gamma}] \left( \frac{\partial u}{\partial y} + \delta^2 \frac{\partial v}{\partial x} \right),$$

$$\tau_{yy} = -2\delta[1 + We \dot{\gamma}] \frac{\partial v}{\partial y}, \quad ,$$

$$\dot{\gamma} = \left[ 2\delta^2 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} + \delta^2 \frac{\partial v}{\partial x} \right)^2 + 2\delta^2 \left( \frac{\partial v}{\partial y} \right)^2 \right]^{\frac{1}{2}}$$

and

$$M^2 = aB_0 \sqrt{\frac{\sigma}{\eta_0}} \text{ is the Hartmann number.}$$

With the lubrication approach and neglecting the terms of order  $\delta$  and  $Re$ , we get

fixed frame to wave frame are as follows  $x = X - ct, y = Y, u = U - c, v = V$  (6.2.2)

Where  $(u, v)$  and  $(U, V)$  are velocity components in the wave and laboratory frames respectively.

$\eta_\infty$  - The infinite shear rate viscosity

$\rho$  - The density

$B_0$  - The magnetic field strength and

$\sigma$  - The electrical conductivity.

Introducing the non-dimensional variables define

$$\bar{x} = \frac{x}{\lambda}, \quad \bar{y} = \frac{y}{a}, \quad \bar{u} = \frac{u}{c}, \quad \bar{v} = \frac{v}{c\delta}, \quad \delta = \frac{a}{\lambda}, \quad \bar{p} = \frac{pa^2}{\eta_0 c \lambda}, \quad \phi = \frac{b}{a}$$

$$h = \frac{H}{a}, \quad \bar{t} = \frac{ct}{\lambda}, \quad \tau_{xx} = \frac{\lambda}{\eta_0 c} \tau_{xx}, \quad \bar{\tau}_{xy} = \frac{a}{\eta_0 c} \tau_{xy}, \quad \tau_{yy} = \frac{\lambda}{\eta_0 c} \tau_{yy},$$

$$Re = \frac{\rho ac}{\eta_0}, \quad We = \frac{\Gamma c}{a}, \quad \bar{\gamma} = \frac{\dot{\gamma} a}{c}, \quad \bar{q} = \frac{q}{ac} \quad (6.2.9)$$

into the Equations (6.2.5) - (6.2.8), reduce to (on ignoring the bars)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (6.2.10)$$

$$Re \delta \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} - \delta^2 \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_{xy}}{\partial y} - M^2 (u + 1) \quad (6.2.11)$$

$$Re \delta^3 \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} - \delta^2 \frac{\partial \tau_{xy}}{\partial y} - \delta \frac{\partial \tau_{yy}}{\partial y} \quad (6.2.12)$$

Where

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left\{ \left[ 1 + We \frac{\partial u}{\partial y} \right] \frac{\partial u}{\partial y} \right\} - M^2 (u + 1) \quad (6.2.13)$$

$$\frac{\partial p}{\partial y} = 0 \quad (6.2.14)$$

From Eq. (6.2.13) and (6.2.14), we get

$$\frac{dp}{dx} = \frac{\partial^2 u}{\partial y^2} + We \frac{\partial}{\partial y} \left[ \left( \frac{\partial u}{\partial y} \right)^2 \right] - M^2 (u + 1) \quad (6.2.15)$$

The corresponding non-dimensional slip boundary conditions in the wave frame are given by

$$u + \beta \left[ \frac{\partial u}{\partial y} + We \left( \frac{\partial u}{\partial y} \right)^2 \right] = -1 \text{ at } y = h = 1 + \phi \cos 2\pi x \quad (6.2.16)$$

$$\frac{\partial u}{\partial y} = 0 \text{ at } y = 0 \quad (6.2.17)$$

The volume flow rate  $Q$  in a wave frame of reference is given by

$$q = \int_0^h u dy \quad (6.2.18)$$

The immediate flow  $Q(X, t)$  in the laboratory frame is

$$Q(X, t) = \int_0^h U dY = \int_0^h (u+1) dy = q + h \quad (6.2.19)$$

The time averaged flux  $\bar{Q}$  over one period  $T\left(\frac{\lambda}{c}\right)$  of the peristaltic wave is given by

$$\bar{Q} = \frac{1}{T} \int_0^T Q dt = q + 1 \quad (6.2.20)$$

Observe that, as  $M \rightarrow 0$  and  $\beta \rightarrow 0$  our results coincide with the results of Shapiro et al [65].

### 6.3 SOLUTION

Since Equation (6.2.15) is a non-linear differential equation, it is not possible to obtain closed form solution. Therefore we employ regular perturbation to find the solution.

For perturbation solution, we consider  $u, p$  and  $q$  as follows

$$u = u_0 + We u_1 + O(We^2) \quad (6.3.1)$$

$$\frac{dp}{dx} = \frac{dp_0}{dx} + We \frac{dp_1}{dx} + O(We^2) \quad (6.3.2)$$

$$q = q_0 + We q_1 + O(We^2) \quad (6.3.3)$$

Making use of these in Equations. (6.2.15) - (6.2.17), we attain

#### 6.3.1. System of order $We^0$

$$\frac{dp_0}{dx} = \frac{\partial^2 u_0}{\partial y^2} - M^2 (u_0 + 1) \quad (6.3.4)$$

and the respective boundary conditions are

$$u_0 + \beta \frac{\partial u_0}{\partial y} = -1 \text{ at } y = h \quad (6.3.5)$$

$$\frac{\partial u_0}{\partial y} = 0 \text{ at } y = 0 \quad (6.3.6)$$

#### 6.3.2. System of order $We^1$

$$\frac{dp_1}{dx} = \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial}{\partial y} \left[ \left( \frac{\partial u_0}{\partial y} \right)^2 \right] - M^2 u_1 \quad (6.3.7)$$

and the respective boundary conditions are

$$u_1 + \beta \frac{\partial u_1}{\partial y} + \beta \left( \frac{\partial u_0}{\partial y} \right)^2 = 0 \text{ at } y = h \quad (6.3.8)$$

$$\frac{\partial u_1}{\partial y} = 0 \text{ at } y = 0 \quad (6.3.9)$$

#### 6.3.3. Solution for system of order $We^0$

Solving Eq. (6.3.4) via the border line (boundary) conditions (6.3.5) and (6.3.6), we attain

$$u_0 = \frac{1}{M^2} \frac{dp_0}{dx} \left[ \frac{\cosh My}{A_1} - 1 \right] - 1 \quad (6.3.10)$$

Where  $A_1 = \cosh Mh + \beta M \sinh Mh$ .

The volume flow rate  $q_0$  is given by

$$q_0 = \frac{dp_0}{dx} \frac{[\sinh Mh - Mh A_1]}{A_1 M^3} - h \quad (6.3.11)$$

From Eq. (6.3.11), we have

$$\frac{dp_0}{dx} = \frac{(q_0 + h) M^3 A_1}{(\sinh Mh - Mh A_1)} \quad (6.3.12)$$

#### 6.3.4. Solution for system of order $We^1$

Substituting Equation (6.3.10) in the Eq. (6.3.7) and solving the Eq. (6.3.7), by means of the border line conditions (6.3.8) and (6.3.9), we attain

$$u_1 = \frac{1}{M^2} \frac{dp_1}{dx} \left[ \frac{\cosh My}{A_1} - 1 \right] + \frac{2}{3} \left( \frac{dp_0}{dx} \right)^2 \left[ A_2 \cosh My + A_1 \sinh My - \frac{A_1}{2} \sinh 2My \right]$$

(6.3.13)

Where

$$A_2 = -\sinh Mh + \frac{1}{2} \sinh 2Mh - \beta M \cosh Mh + \beta M \cosh 2Mh - \frac{3}{2} \beta M \sinh^2 Mh$$

The volume flow rate  $q_1$  is given by

$$q_1 = \frac{1}{A M^3} \frac{dp_1}{dx} [\sinh Mh - Mh A_1] + \frac{2}{3} \frac{A_3}{A^3 M^4} \left( \frac{dp_0}{dx} \right)^2 \quad (6.3.14)$$

Where

$$A_3 = A_2 \sinh Mh + A_1 \cosh Mh - \frac{A_1}{4} \cosh 2Mh - \frac{3}{4} A_1^2$$

From Eq. (6.3.14) and (6.3.12), we have

$$\frac{dp_1}{dx} = \left[ q_1 - \frac{2}{3} \frac{A_3}{A^3 M^4} \left( \frac{dp_0}{dx} \right)^2 \right] A M^3 / (\sinh Mh - Mh A_1)$$

(6.3.15)

Making use of Equations (6.3.12) and (6.3.15) in the Eq. (6.3.2) and also the relation

$$\frac{dp_0}{dx} = \frac{dp}{dx} - We \frac{dp_1}{dx}$$

by neglecting terms larger than  $O(We)$ , we acquire

$$\frac{dp}{dx} = \left[ \frac{(q+h) A M^3}{(\sinh Mh - Mh A_1)} - We \frac{2 A_3 (q+h)^2 M^5}{3 (\sinh Mh - Mh A_1)^3} \right] \quad (6.3.16)$$

The non-dimensional hike (rise) in pressure for unique wavelength in the wave frame is defined as

$$\Delta p = \int_0^1 \frac{dp}{dx} dx \quad (6.3.17)$$

### 6.4. RESULTS AND DISCUSSIONS

Fig. 6.2 illustrates the fluid flow pattern for a preference of values of the Weissenberg number

$We$ . The graph is plotted by considering pressure gradient  $\frac{dp}{dx}$  along Y direction and Noted the variations in the pressure gradient along the X axis for the fixed values of  $\phi = 0.6$ ,  $M = 1$  and  $\beta = 0.1$ . We can make out from the graph that the  $\frac{dp}{dx}$  increases when there is a rise in  $We$ .

**Fig. 6.3** illustrates the fluid flow pattern for a preference of values of the slip parameter  $\beta$ . The graph is plotted by considering pressure gradient  $\frac{dp}{dx}$  along Y direction and Noted the deviations in the pressure gradient along the X axis for the fixed values of  $\phi = 0.6$ ,  $M = 1$  and  $We = 0.02$ . We can make out from the graph that the  $\frac{dp}{dx}$  decreases when there is a rise in  $\beta$ .

**Fig. 6.4** illustrates the fluid flow pattern for a preference of values of the Hartmann number  $M$ . The graph is plotted by considering pressure gradient  $\frac{dp}{dx}$  along Y direction and Noted the deviations in the pressure gradient along the X axis for the fixed values of  $\phi = 0.6$ ,  $We = 0.02$  and  $\beta = 0.1$ . We can make out from the graph that the  $\frac{dp}{dx}$  increases when there is a rise in  $M$ .

**Fig. 6.5** illustrates the fluid flow pattern for a preferred of values of the Amplitude Ratio  $\phi$ . The graph is plotted by considering pressure gradient  $\frac{dp}{dx}$  along Y direction and Noted the deviations in the pressure gradient along the X axis for the fixed values of  $We = 0.02$ ,  $M = 1$  and  $\beta = 0.1$ . We can make out from the graph that the  $\frac{dp}{dx}$  increases when there is a rise in  $\phi$ .

**Fig. 6.6** illustrates the deviation in the pressure rise  $\Delta p$  against time averaged volume flow rate  $\bar{Q}$ .  $\Delta p$  Has been taken along Y axis and  $\bar{Q}$  is taken along X-axis. Graph is obtained for distinct values of the Weissenberg number  $We$ . When  $\phi = 0.6$ ,

$\beta = 0.1$  and  $M = 1$ . Where and when the flow takes place is noted. In pumping ( $\Delta p > 0$ ), free pumping ( $\Delta p = 0$ ) and in the co pumping ( $\Delta p < 0$ ) regions the volume flow rate  $\bar{Q}$  rises up when there is a rise in  $We$ .

**Fig. 6.7** illustrates the deviation in the pressure rise  $\Delta p$  against time averaged volume flow rate  $\bar{Q}$ .  $\Delta p$  Has been taken along Y axis and  $\bar{Q}$  is taken along X-axis. Graph is plotted for distinct values of the Slip parameter  $\beta$ . When  $\phi = 0.6$ ,  $We = 0.02$  and  $M = 1$ . Where and when the flow takes place is noted. In pumping ( $\Delta p > 0$ ) and free pumping ( $\Delta p = 0$ ) regions the volume flow rate  $\bar{Q}$  decreases when there is a hike in  $\beta$ . even if, in the co pumping ( $\Delta p < 0$ ) region the volume flow rate  $\bar{Q}$  rises up when there is a rise in  $\beta$ .

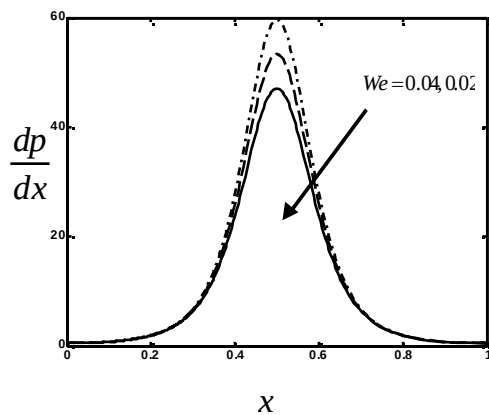
**Fig. 6.8** Reveals the deviation in the pressure rise  $\Delta p$  against time averaged volume flow rate  $\bar{Q}$ .  $\Delta p$  Has been taken along Y axis and  $\bar{Q}$  is taken along X-axis. Graph is plotted for distinct values of the Hartmann number  $M$ . When  $\phi = 0.6$ ,  $We = 0.02$  and  $\beta = 0.1$ . Where and when the flow takes place is noted. In pumping ( $\Delta p > 0$ ) region the volume flow rate  $\bar{Q}$  increases when there is a hike in  $M$ . Even if, in both the co pumping ( $\Delta p < 0$ ) region and free pumping ( $\Delta p = 0$ ) region, the volume flow rate  $\bar{Q}$  goes down when there is a rise in  $M$ .

**Fig. 6.9** Reveals the deviation in the pressure rise  $\Delta p$  against time averaged volume flow rate  $\bar{Q}$ .  $\Delta p$  Has been taken along Y axis and  $\bar{Q}$  is taken along X-axis. Graph is plotted for distinct values of the Amplitude ratio  $\phi$ . When  $M = 1$ ,  $We = 0.02$  and  $\beta = 0.1$ . Where and when the flow takes place is noted. In both pumping ( $\Delta p > 0$ ) region and free pumping ( $\Delta p = 0$ ) region, the volume flow rate  $\bar{Q}$  increases when there is a hike in  $\phi$ . Even if, in co pumping

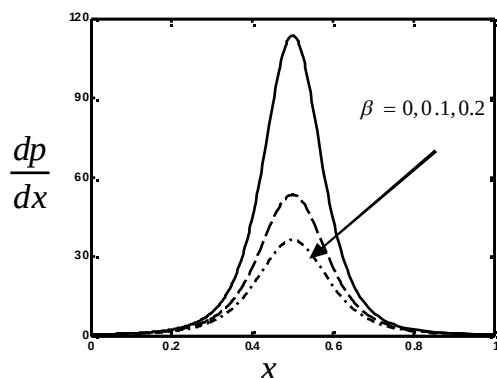
( $\Delta p < 0$ ) region, the volume flow rate  $\bar{Q}$  goes down when there is a rise in  $\phi$ .

## CONCLUSION

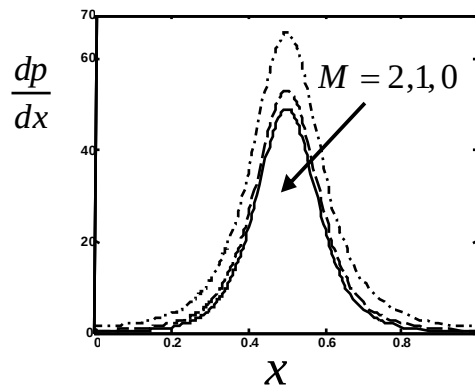
In this chapter, we did an experiment on the influence of slip and magnetic field on Williamson fluid flow in a planar channel with peristalsis, with aid of low Reynolds number and long wavelength consideration. We found the velocity of the wave in a wave frame of reference. The perturbation solution is found for velocity field and pressure gradient per one wavelength. It is also found that the axial pressure gradient and time-averaged volume flow rate, in the pumping region rises up if there is hike in Weissenberg number  $We$ , Hartmann number  $M$  and amplitude ratio  $\phi$  although they moves down with hike in slip parameter  $\beta$ . Further, it is found that the “pumping is more for Williamson fluid than that of Newtonian fluid”.



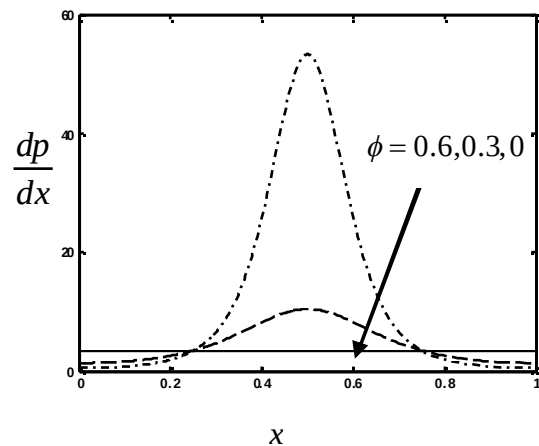
**Fig. 6.2.**Pressure gradient  $\frac{dp}{dx}$  with the axis of the flow is plotted for  $We$  with  $\phi = 0.6$ ,  $\beta = 0.1$  and  $M = 1$ .



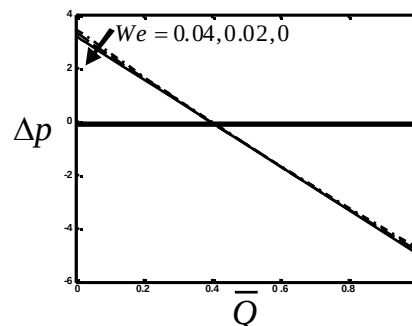
**Fig. 6.3.**Pressure gradient  $\frac{dp}{dx}$  with the axis of the flow is plotted for  $\beta$  with  $\phi = 0.6$ ,  $We = 0.02$  and  $M = 1$ .



**Fig. 6.4.**Pressure gradient  $\frac{dp}{dx}$  with the axis of the flow is plotted for  $M$  with  $\phi = 0.6$ ,  $\beta = 0.1$  and  $We = 0.02$ . For  $M = 2, 1, 0$



**Fig. 6.5.**Pressure gradient  $\frac{dp}{dx}$  with the axis of the flow is plotted for  $\phi$  with  $M = 1$ ,  $\beta = 0.1$  and  $We = 0.02$ .



**Fig. 6.6.**The dissimilarity in pressure rise  $\Delta p$  with flux  $\bar{Q}$  for unlike Values of  $We$  with  $\phi = 0.6$ ,  $\beta = 0.1$  and  $M = 1$

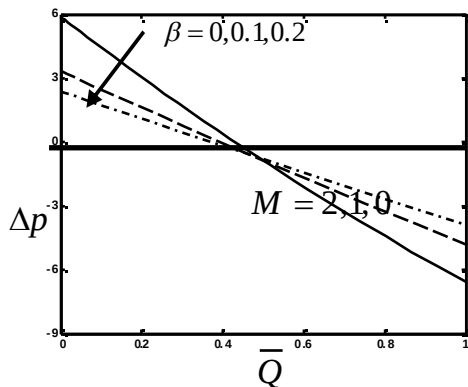


Fig. 6.7. The dissimilarity in pressure rise  $\Delta p$  with flux  $\bar{Q}$  for like Values of  $\beta$  with  $\phi = 0.6$ ,  $We = 0.02$  and  $M = 1$ .

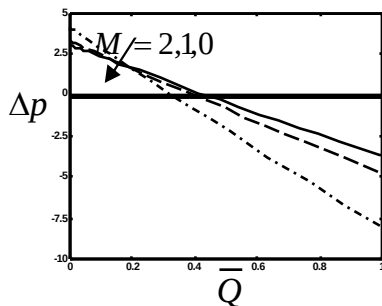


Fig. 6.8. The dissimilarity in pressure rise  $\Delta p$  with flux  $\bar{Q}$  for unlike

Values of  $M$  with  $\phi = 0.6$ ,  $\beta = 0.1$  and  $We = 0.02$ .

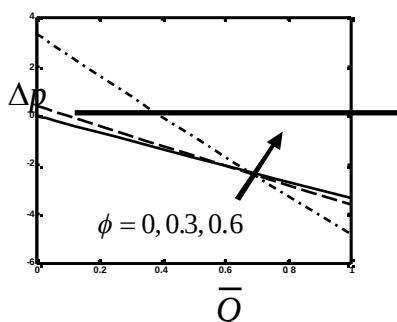


Fig. 6.9. The dissimilarity in pressure rise  $\Delta p$  with flux  $\bar{Q}$  for Unlike Values of  $\phi$  with  $We = 0.02$ ,  $\beta = 0.1$  and  $M = 1$ .

## REFERENCES

- [1] Abd El Hakeem A.E.N and El Misiery, A.E.M. Effects of an endoscope and generalised Newtonian Fluid on peristaltic motion, Appl. Math.Comput. 128(2002), 19-35.
- [2] Abd El Hakeem Abd El Naby, El Misiery. A.E.M, and El Shamy, I. Hydromagnetic flow of fluid with variable viscosity in uniform tube with peristalsis, J. Phys.A: Math. Gen. 36 (2003), 8535-8547.
- [3] Abd El Hakeem A.E.N, El Misiery. A.E.M, and Abd El Kareem, M. F. Separation in flow through peristaltic motion of a Carreau fluid in uniform tube, Physica A, 343(2004), 1-15.
- [4] Abd El Hakeem Abd El Naby, El Misiery. A.E.M, and El Shamy, I. Effects of an endoscope and fluid with variable viscosity on peristaltic motion, Appl. Math.Comput., 158(2004), 497-511.
- [5] Adomian, G. Application of the decomposition method to the Navier-Stokes equations, J. Math.Anal. Appl. 119 (1986), 340-360.
- [6] Adomian, G. Solving frontier problems of physics: the Decomposition method, Kluwer. Acad. Boston, 1994.
- [7] Adomian, G. Explicit solutions of nonlinear partial differential equations, Appl. Math.Comput. 88 (1997), 117-126.
- [8] Afifi, N. A. S. and Gad, N. S. Interaction of peristaltic flow with pulsatile magneto fluid through a porous medium, ActaMechanica, 149(2001), 229-237.
- [9] Agrawal, H.L and Anwaruddin, B. Peristaltic flow of blood in a branch, Ranchi University Math. J. 15(1984), 111-121.
- [10] Akbar, N. S., Nadeem, S. and Lee, C. Peristaltic flow of a Prandtl fluid model in an asymmetric channel, Int. J. Phys. Sci., 7(5)(2012), 687-695.
- [11] Ali, N. and Hayat, T. Peristaltic motion of a Carreau fluid in an asymmetric channel, Appl. Math.Comput., 193(2007), 535-552.

- [12]Ali, N., Hussain, Q., Hayat, T. and Asghar, S. Slip effects on the peristaltic transport of MHD fluid with variable viscosity, *Physics Letters A*, 372 (2008) 1477-1489.
- [13]Ali, N. and Hayat, T. Peristaltic flow of a micropolar fluid in an asymmetric channel, *Computers and Mathematics with Applications*, 55, 589-608 (2008).
- [14]Ali, N., Wang, Y., Hayat, T. and Oberlack, M. Slip effects on the peristaltic flow of a third grade fluid in a circular cylindrical tube, *J. Appl. Mech.*, 76(2009), 1-10.
- [15]Barnes, H.A., Hutton, J.F. and Walters, K. n introduction to Rhyology, Elsevier, Amsterdam, 1989.
- [16]Beavers, G.S. and Joseph, D.D. Boundary conditions at a naturally permeable wall, *J. Fluid Mech.*, 30 (1967), 197-207.
- [17]Bohme, G., and Friedrich, R. Peristaltic flow of viscoelastic liquids, *J. Fluid Mech.*, 128 (1983), 109-122.
- [18]Burns, J.C. and Parkes, T. Peristaltic motion, *J. Fluid Mech.*, 29(1967), 731-743.
- [19]De Vries, K., Lyons, E.A., Ballard, J., Levi, C.S. and Lindsay, D.J. Contractions of the inner third of the myometrium, *Ame.J.Obstetrics Gynecol.*, 162(1990), 679-682.
- [20].Derek, C., Tretheway, D.C. and Meinhart, C.D. *Phys. Fluids*, 14(2002) L9.
- [21]Ebaid, A. A new numerical solution for the MHD peristaltic flow of a bio-fluid with variable viscosity in a circular cylindrical tube via Adomian decomposition method, *Physics Letters A* 372 (2008), 5321-5328.
- [22]Eldabe, N. T., Elghazy, E. M. and Ebaid, A. Closed form solution to a second order boundary value problem and its application in fluid mechanics, *Phys. Lett. A* 363 (2007), 257-259.
- [23]El Misery, A. M., El Shehawey, E. F. and Hakeem, A. A. Peristaltic motion of an incompressible generalized Newtonian fluid in a planar channel, *J. Phys. Soc. Jpn.*, 65 (1996), 3524-3529.
- [24]El Shehawey, E.F., El Misery, A. M., and Hakeem, A. A. Peristaltic motion of generalized Newtonian fluid in a non-uniform channel, *J. Phys. Soc. Jpn.*, 67 (1998), 434-440.
- [25]El Shehawey, E.F., Mekheimer, Kh. S., Kaldas, S. F. and Afifi, N. A. S. Peristaltic transport through a porous medium, *J. Biomath.*, 14 (1999).
- [26]El Shehawey, E.F., Sobh, A.M.F. and Elbarbary, E.M.E. Peristaltic motion of a generalized Newtonian fluid through a porous medium, *Phys. Soc. Jpn.*, 69(2000), 401- 407.
- [27].El Shehawey, E.F. and Husseny, S.Z.A. Effects of porous boundaries on peristaltic transport through a porous medium, *ActaMechanica*, 143(2000), 165-177.
- [28]El Shehawey, E.F. and EL SEBAEI, W. Peristaltic transport in a cylindrical tube through a porous medium, *Int.J. Math.Math. Sci.*, 24 (2000), 217-230.
- [29]Elshahed, M. and Haroun, M. H., Peristaltic transport of Johnson-Segalman fluid under the effect of a magnetic field, *Mathematical Problems in Engineering*, 6(2005), 663-677.
- [30]El Shehawey, E.F., Eldabe, N.T., Elghazy, E.M. and EBAID, A. peristaltic transport in an asymmetric channel through a porous medium, *Appl. Math.Comput.* 182(2006), 140-150.
- [31]El-Shehawey, E.F., El-Dabe, N.T. and El-Desoky, I. M. Slip effects on the Peristaltic flow of a Non-Newtonian Maxwellian Fluid, *Acta Mech.*, 186(2006), 141-159.
- [32]Eytan, O. and Elad, D. Analysis of Intra – Uterine fluid motion induced by uterine contractions, *Bull. Math.Bio.*, 61(1999), 221-238.
- [33]Eytan, O., Jaffa, A.J. and Elad, D. Peristaltic flow in a tapered channel : application to embryo transport within the uterine cavity, *Med. Engng. Phys.*, 23(2001), 473-482.
- [34]Fauci, Lisa J. Peristaltic pumping of solid particles, *Comp. Fluids*, 21 No.4(1992), 583-592.
- [35]Haroun, M. H. Effect of Deborah number and phase difference on peristaltic transport of a third – order fluid in an asymmetric channel, *Communications in Nonlinear Science and Numerical Simulation*, 12(2007),1464-1480.
- [36]Hayat, T., Wang, Y., Siddiqui, A.M., Hutter, K. and Asghar, S. Peristaltic transport of a third-order fluid in a circular cylindrical tube, *Math. Models & Methods in Appl. Sci.*, 12(2002), 1691-1706.
- [37]Hayat, T., Wang, Y., Siddiqui, A.M., and Hutter, K. Peristaltic motion of a Johnson – Segalman fluid in a planar channel, *Math.Problems in Engng.* 1 (2003), 1-23.
- [38].Hayat, T and Ali, N. Peristaltically induced motion of a MHD third grade fluid in a deformable tube, *Physica A: Statistical Mechanics and its Applications*, 370(2006), 225-239.
- [39]Hayat, T., Ali, N., Asghar, S. and Siddiqui, A. M. Exact peristaltic flow in tubes with an endoscope, *Appl. Math.Comput.* 182 (2006) 359-368.
- [40]Hayat, T., Afsar, A., Khan, M. and Asghar, S. Peristaltic transport of a third order fluid under the effect of a magnetic field, *Computers and Mathematics with Applications*, 53(2007), 1074-1087.

- [41] Hayat, T., Ali, N. and Asghar, S. Hall effects on peristaltic flow of a Maxwell fluid in a porous medium, *Phys. Letters A*, 363(2007), 397-403.
- [42] Hayat, T., Khan, M., Siddiqui, A. M. and Asghar, S. Non-linear peristaltic flow of a non-Newtonian fluid under the effect of a magnetic field in a planar channel, *Communications in Nonlinear Science and Numerical Simulation*, 12(2007), 910-919.
- [43] Hayat, A. and Ali, N. Peristaltic motion of a Jeffrey fluid under the effect of a magnetic field in a tube, *Communications in Nonlinear Science and Numerical Simulation*, 13(2008), 1343-1352.
- [44] Hayat, T., Naveed Ahmed and Sajid, M. Analytic solution for MHD flow of a third order fluid in a porous channel, *Journal of Computational and Applied Mathematics*, 214 (2008), 572 – 582.
- [45] Hayat, T., Ahmad, N. and Ali, N. Effects of an endoscope and magnetic field on the peristalsis involving Jeffrey fluid, *Communications in Nonlinear Science and Numerical Simulation*, 13(2008), 1581-1591.
- [46] Hayat, T. Asfar, A. and Ali, N. Peristaltic transport of a Johnson-Segalman fluid in an asymmetric channel, *Mathematical and Computer Modelling*, 47(2008), 380-400.
- [47] Jaffrin, M.Y. and Shapiro, A.H. Peristaltic Pumping, *Ann. Rev. Fluid Mech.*, 3(1971), 13-36.
- [48] Jaffrin, M.Y. Inertia and streamline curvature effects on peristaltic pumping, *Int. J. Engng. Sci.*, 11(1973), 681-699.
- [49] Kwang – Hua Chu, W. and Fang, J. Peristaltic transport in a slip flow, *Eur. Phys., J. B.*, 16(2000), 543-547.
- [50] Latham, T.W. Fluid motions in peristaltic pump, M.S. Thesis, MIT, Cambridge, Massachusetts, 1966.
- [51] Li, M. and Brasseur, J.B. Non-steady peristaltic transport in finite-length tubes, *J. Fluid Mech.*, 248(1993), 129-151.
- [52] Mekheimer, K.H.S. And Al-Arabi, T.H. Nonlinear peristaltic transport of MHD flow through a porous medium, *Int. J. Math. Math. Sci.*, 26(2003), 1663-1682.
- [53] Mishra, M. and Ramachandra Rao, A. Peristaltic transport of a Newtonian fluid in an asymmetric channel, *Z. Angew. Math. Phys. (ZAMP)*, 54(2003), 532-550.
- [54] Mekheimer K.H.S. Non linear peristaltic transport through a porous medium in an inclined planar channel, *J. Porous Media*, 6(2003), 190-202.
- [55] Mekheimer, K.H.S. Peristaltic flow of blood under effect of a magnetic field in a non uniform channel, *Appl. Math. Comput*, 153 (2004), 763-777.
- [56] Nadeem, S. and Akram, S. Peristaltic flow of a Williamson fluid in an asymmetric channel, *Comm. Non linear sic. Numer. Simul.*, 15 (2010), 1705–1716.
- [57] Noreen, S., Hayat, T. and Alsaedi, A. Study of slip and induced magnetic field on the peristaltic flow of pseudoplastic fluid, *International Journal of Physical Sciences*, Vol. 6(36)(2011), 8018-8026.
- [58] Pozrikidis, C. A study of peristaltic flow, *J. Fluid Mech.*, 180(1987), 515-527.
- [59] Radhakrishnamacharya, G. Long wavelength approximation to peristaltic motion of a power law fluid, *Rheol. Acta.*, 21(1982), 30-35.
- [60] Raju, K.K. and Devanathan, R. Peristaltic motion of a non-Newtonian fluid part –I, *Rheol. Acta.*, 11(1972), 170-178.
- [61] Rao, A. R. and Mishra, M. Non-linear and curvature effects on peristaltic flow of a viscous fluid in an asymmetric channel, *Acta Mech.* 168(2004), 35-59.
- [62] Rath, H.J. *Peristaltische Stromungen*, Springer, 1980.
- [63] Reese, G.W., and Rath, H.J. A model of a combined porous peristaltic pumping system, In *Physiological Fluid Dynamics II*, Proc. 2<sup>nd</sup> International conference on physiological fluid dynamics, IIT Madras, L.S. Srinath and M. Singh, eds., Tata McGraw-Hill, New Delhi, 1987, 190-195.
- [64] Shapiro, A.H. Pumping and retrograde diffusion in peristaltic waves, in: *Proceedings of the workshop I Ureteral Reflux in Children*, (1967), 109.
- [65] Shapiro, A.H., Jaffrin, M.Y. and Weinberg, S.L. Peristaltic pumping with long wavelengths at low Reynolds number, *J. Fluid Mech.* 37(1969), 799-825.
- [66] Shukla, J.B. and Gupta, S.P. Peristaltic transport of a power-law fluid with variable consistency, *Trans. ASME J. Biomech. Engng.*, 104(1982), 182-186.
- [67] Siddiqui, A.M., Provost, A. and Schwarz, W.H. Peristaltic pumping of a second-order fluid in a planar channel, *Rheol. Acta*, 30(1991), 249-262.
- [68] Siddiqui, A.M., Provost, A. and Schwarz, W.H. Peristaltic pumping of a third-order fluid in a planar channel, *Rheol. Acta*, 32(1993), 47-56.
- [69] Siddiqui, A.M. and Schwarz, W.H. Peristaltic flow of a second order fluid in tubes, *J. Non-Newtonian Fluid Mech.*, 53(1994), 257-284.
- [70] Siddiqui, A.M., Hayat, T. and Khan, M. Magnetic fluid model induced by peristaltic waves, *J. Phys. Soc. Jpn.*, 73(2004), 2142-2147.



- [71]Srivastava, L.M. and Srivastava, V.P, Peristaltic transport of blood: Casson model II, J. Biomech., 17(1984), 821-829.
- [72]Srivastava, L.M., and Srivastava, V.P. Peristaltic transport of a non – Newtonian fluid: application to the vas deferens and small intestine, Annals Biomed. Engng., 13(1985), 137-153.
- [73] Srivastava, L. M. Peristaltic transport of a Casson fluid, Nig. J. Sci. Res. 71-82, (1987).
- [74]Srivastava, L.M. and Srivastava, V.P. Peristaltic transport of power law fluid: application to the ductus of efferentus of the reproductive tract, Rheol. Acta., 27(1988), 428-433.
- [75]Srinivasacharya, D., Mishra, M. and Ramachandra Rao, A. Peristaltic pumping of a micro polar fluid in a tube ,ActaMechanica, 161(2003), 165-178.
- [76]Sreenadh, S., Narahari, M. and Ramesh Babu, V. Effect of yield stress on peristaltic pumping of Non-Newtonian fluids in a channel, International Symposium on advances in fluid Mechanics, UGC-Centre for Advanced Studies in Fluid mechanics, Bangalore University, June-2004, 234-247
- [77]Sud, V.K., Seshon, G.S. and Mishra, R.K. pumping action on blood flow by a magnetic field, Bull. Math. Biol. 39 (1977) 385 – 390.
- [78] Subba Reddy, M. V., Mishra, M., Sreenadh, S. and Ramachandra Rao, A. Influence of lateral walls on peristaltic pumping in a rectangular duct, Trans. ASME, Journal of Fluids Engineering, 127(2005), 824 –827.
- [79]Subba Reddy, M. V., Ramachandra Rao, A. and Sreenadh, S. Peristaltic motion of a power law fluid in an asymmetric channel, Int. J. Non-Linear Mech., 42 (2007), 1153-1161.
- [80]Subba Reddy, M. V. and Gangadhar, K. Non-linear peristaltic motion of a Carreau fluid under the effect of a magnetic field in an inclined planar channel, International Journal of Dynamics of Fluids, Vol. 6(1)(2010), 63-76.
- [81]Subba Reddy, M. V., Jayarami Reddy, B. and Prasanth Reddy, D. Peristaltic pumping of Williamson fluid in a horizontal channel under the effect of magnetic field, International Journal of Fluid Mechanics, Vol. 3(1)(2011), 89-109.
- [82] Subba Reddy, M.V., Jayarami Reddy, B., Chandrasekhar Babu, S. and B. Jayaraj, Peristaltic transport of a Johnson - Segalman fluid through a
- [85] Sudhakar Reddy, M., Subba Reddy, M. V., Jayarami Reddy, and B. And S. Ramakrishna, Effect of variable viscosity on the peristaltic flow of a Jeffrey fluid in a uniform tube, Advances in Applied Science Research, 3 (2)(2012), 900-908.
- [86]Takabatake, S. Ayukawa, K. and Mori, A. Peristaltic pumping in circular tubes: A numerical study of fluid transport and its efficiency, J.Fluid Mech., 193(1988), 267-283.
- [87] Takahashi, T., Ogoshi, H., Miyamoto, K. and Yao, M.L. Viscoelastic properties of commercial plain yoghurts and trial foods for swallowing disorders, Rheology, 27(1999), 169–172.
- [88]Tsiklauri, D. and Beresnev, I. Non-Newtonian effects in the peristaltic flow of a Maxwell fluid, Phys. Rev. E, 64(2001), 036303.
- [89] Vajravelu, K., Sreenadh, S. and Ramesh Babu, V. Peristaltic transport of a Hershel-Bulkley fluid in an inclined tube, Int. J. Non linear Mech. 40(2005) 83-90.
- [90] Vajravelu, K. Sreenadh, S. and Ramesh Babu, V. Peristaltic pumping of a Hershel-Bulkley fluid in a channel, Appl. Math. and Computation, 169(2005), 726-735.
- [91]Vajravelu, K., Radhakrishnamacharya, G. and Radhakrishnamurthy, V. Peristaltic flow and heat transfer in a vertical porous annulus, with long wave approximation, Int. J. Non-Linear Mech., 42(2007), 754-759.
- [92] Yin, F. and Fung, Y.C. Peristaltic waves in circular cylindrical tubes, Trans. ASME J. Appl. Mech. 36(1969), 579-587. porous medium in a channel, International Journal of Mathematical Archive, 2(2011), 646 – 658.
- [83] Subba Reddy, M. V., Jaya Rami Reddy, B., Sudhakar Reddy, M. and Nagendra, N. Long wavelength approximation to MHD peristaltic flow of a Bingham fluid through a porous medium in an inclined channel, International Journal of Dynamics of Fluids, 7(2)(2011), 157-170.
- [84] Subba Reddy, M. V., Jayarami Reddy, B., Nagendra, N. and Swaroopa, B. Slip effects on the peristaltic motion of a Jeffrey fluid through a porous medium in an asymmetric channel under the effect magnetic field, Journal of Applied Mathematics and Fluid Mechanics, Vol. 4(1)(2012), 59-72.