THINK INDIA JOURNAL

ISSN:0971-1260
Vol-22- Issue-14-December-2019

Combinatorical Applications of Rook Polynomials in Various Fields

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ABSTRACT

In this paper, we studied the game of chess, the rook and its movements to capture pieces in the same row or column as the rook. With this idea we applied it to combinatorial problems which involve permutation with Forbidden positions. By applying generating functions and n × m arrays to construct rook polynomials in a combinatorial way.

Keywords—r-arrangement, combinatorial structures, Chess movements.

I. INTRODUCTION

The problem of counting arrangements of objects where there are restrictions in some of the positions in which they can be placed has been addressed by different authors with applications such as; need to match applicants to jobs, where some of the applicants cannot hold certain jobs, or need to pair up artists, but some of the artists cannot be paired up with some of the artists [2]. Now, to address this type of problems where there is need to find the number of arrangements with "Forbidden positions" has been addressed so many years ago. These can be traced back to the early eighteenth century when the French mathematician PIERRE DE MONTMORT studied the problem des rencontres (the matching problem) [6]. In this paper our focus is on forbidden positions in a chess board, a rook has the ability to capture pieces in the same row or column as the rook. We use this idea and apply it to combinatorial problem that involves permutation with forbidden positions.

II. BASIC DEFINITIONS

Rook: A rook is a chess piece that moves horizontally or vertically and can take (or capture) a piece if that piece rests on a square in the same row or column as the rook [2, 4, 5].

a. Board: A board B is an n×m array of n rows and m columns. When a board has a darkened square, it is said to have a forbidden position.

b. Rook polynomial: A rook polynomial on a board B, with forbidden positions is denoted as R (x, B),

Where R(x, B), has coefficients $r_k(B)$ representing the number of non-capturing rooks on board B. Clearly, we have just one way of not placing a rook. Thus $r_0(B) = 1$

c. A board B with forbidden positions, is said to be disjoint if the board can be decomposed into two sub-boards B_i: i= 1 and 2 such that, neither B_1 nor B_2 share the same row or column. [5] f

1	2		
3	4		
		5	6
		7	8

Fig i

1			
2	3		

ISSN:0971-1260
Vol-22- Issue-14-December-2019

	4	5	6

Fig ii

with columns. This allows us to attempt to make non-disjoint boards into disjoint boards. Non-taking rooks is to enumerate the number of ways of placing k rooks on a chessboard such that no rook will be captured by any other rook.

III. THEOREMS

Theorem 1 (Principle of inclusion-exclusion (PIE). ([4, 6])

Theorem 2 If B is a board of numbered squares that decomposes into two disjoint sub-boards i: = 1, 2, then B_i:i =1, 2 then R(x, B)= R(x, B1)R(x, B2). [4]

Theorem 3 The number of ways to place n non attacking rooks on an n × n board with forbidden positions is

$n! - r1(n - 1)! + r2(n - 2)! - + \cdot\cdot\cdot + (-1)^n r_n 0!$, where the numbers r_k are the coefficients of the rook polynomial for the forbidden positions.

IV. APPLICATIONS

The admission officer selected the following students (Ada (A), Chinedu (C), Fonyuy (F), Gabche (G) and Kinyuy (K) with highest scores during the 2016/2017 JAMB examinations. To offer this students admission, the V.C. of Chukwuemeka Odimuegwu Ojukwu University, Uli, then requested for the number of ways it will take the University to offer admission to the students in any of the following faculties; Biological Science (B), Engineering (E), Law (L), Management (M) and Physical Science, provided the students meet the faculty requirements. It follows that; a. the Faculty of Biological Science (B) can't admit Fonyuy (F) and Gabche (G) b. the Faculty of Engineering (E) can't admit Ada (A) and Chinedu (C), c. the Faculty of Law (L) can't admit Fonyuy (F) d. the Faculty of Management (M) can't admit Fonyuy (F), e. the Faculty of Physical Science can't admit Chinedu (C), and Kinyuy (K) In how many ways can the admission officer admit the selected students?

Clearly, Fig i is disjoint while Fig ii is not. Boards are invariant, they can be rearranged by swapping rows with rows or by swapping columns

V. ANALYSIS

The combinatorial analyst employed the generating functions and n × m arrays to construct rook polynomials R(x, B) = $1 + 8x + 21x^2 + 20x^3 + 6x^4$ in a combinatorial way. To determine the number of possible admission positions for the five students into the various faculties. We now applied Theorem **1 Principle of inclusion and exclusion** to obtain the numbers of possible admission positions that could be allocated to this particular set of students with Forbidden position.

VI. REPORT

The Admission Officer informs the V.C. that twenty admission allocations can be given to the special students with high performance in the 2016/2017 JAMB examinations.

VII. CONCLUSION

This paper presents Applications of Rook polynomial in different fields. Polynomial can be generated by using coding techniques.

VII RECOMMENDATIONS

Further study can be carried out with the bishops on three-dimensional boards. In addition, further studies could also examine what would happen to rook and bishop polynomials by allowing the rook or bishop to attack each other on the chess boards

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ISSN:0971-1260
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