

Cosmic Acceleration In An Anisotropic Background In The Presence of Bulk Viscosity

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Abstract

In this paper, we investigated a cosmological situation that proposes a variation law in which a simple linear function of Hubble's parameter seems like a deceleration parameter that provides the scale factor. We have acquired the cosmological models that shows the universe starts to accelerate and increases exponentially with infinite time t . Hubble parameter gives the fixed value of deceleration parameter which acquires the required results of the field equations of Einstein. This parameter in the model is observed to be time subordinate which shows the development from initial decelerating period to the present accelerating period of expansion and supplies the biggest value and the fastest rate at which the universe is expanding. Model offers dynamically anisotropic expansion to the universe which allows for symmetrical Bianchi metrics.

Keywords: Cosmological Model, Constant deceleration parameter, Field Equations, Linear Equations.

I. INTRODUCTION

Cosmology is analysis of the fundamental systems and movements of our cosmos which concerns with their foundation and growth. It also includes learning about the movements of the heavenly bodies. The universe is not only isotropic but also uniform according to the current condition of growth. In cosmology, the problem of cosmological constant is considered to be one of the greatest problem. In the Einstein's field equations, this cosmological constant problem was primarily specified. This can be done by considering different values for cosmological constant. In recent years, there has been significant and essential evidence about the recognition of Lambda as an Einstein's cosmological constant. A theory of universe combines dynamic degrees of freedom with matter fields. Therefore through development of cosmos and formation of elements, (Λ) reduces to its current slight value as the universe is very old so this constant value is very small.

The field equations of Einstein are a system of simultaneous nonlinear differential equations. In the field equations, physical solutions have been identified in the context of cosmology and Astrophysics. We propose properties of substance or properties of spacetime for solving the field equations which require the metric of corresponding space of Killing, Kramer and Schmutzer [1].

Pavon, D. [2] assumed the scale factor of principle of divergence generates the solution to field equations. The matter of several investigation for the solution of field equations of Einstein is specified by cosmological scale variable $R(t)$. Pavon, D. [2], Holye, F. et al [3], Olson, T.S. et al. [4], Maia, M.D. et al. [5], Silveria, V. et al. [6, 7], Torres, L.F.B. et al. [8], Chen and Wu

[9] investigated cosmological models with cosmological term that are equivalent to scale factor. According to the prior history, they examined Λ as varying $(-2)^{th}$ power of R which is denoted as the scale factor. By deriving the value of Λ , Carvalho, J. C. et al. [10] considers the value of $\Lambda = \alpha R^{-2} + \beta H^2$, in the metric of Robertson – Walker, R is indicated as the scale factor, H is signified as the Hubble parameter and amendable dimensionless parameters are signified as α, β .

In the structure of general relativity, Dirac [11] offered the proposal of variable gravitational constant G and also Lau [12] suggested the variation connecting the variation of G with Λ . By applying this method, number of authors A.-M. M. Abdel-Rahman [13], Berman, M. S. et al. [14], Sister'o, R. F [15], Kalligas, D. et al. [16], Vishwakarma, R. G. et al. [17-19], Pradhan, A., & Otarod, S [20], Singh, C. P., Kumar, S. et al. [21], Singh, J. P. et al. [22] explored Bianchi models. The latest papers of Borges, H. A., & Carneiro, S [23] shows Bianchi form-I model with changing gravitational constant and Λ , as an isotropic and uniform smooth universe occupied with substance and a cosmological term that is equivalent to Hubble parameter H .

The Prime purpose of proposed work is to explore with variable Λ in the model of the perfect fluid which contains matter in the uniform Bianchi form-I, for the stiff matter. Say for stiff matter, the square of the Hubble parameter is proportionate to cosmological term, though we acquired the results of Einstein's equations.

The manuscript leads by the series of sections as mentioned: In section 2, the basic definitions of the models are mentioned. In the division 3, their solution are obtained. In section 4, the conclusion drawn from the results. In section 5, the future research idea is given.

II. THE METRIC AND FIELD EQUATIONS

Line element

$$ds^2 = dt^2 - A^2(t) dx^2 - B^2(t) [dy^2 + dz^2] \dots (1)$$

Defines the spatially uniform and anisotropic Bianchi type-I metric space time.

Here the A, B are metric tensors ($c = 1, 8\pi G = 1$) that denotes cosmic time functions t .

The cosmic matter represented by a perfect fluid of the energy momentum tensor

$$T_{ij} = (\rho + p) v_i v_j + p g_{ij} \dots (2)$$

" p " specifies its pressure, energy density of the cosmic matter is denoted by ρ .

The four velocity vector is denoted by v_i , therefore $v_i v^i = 1$.

$$\text{The equation of state is defined as } p = \omega \rho, 0 \leq \omega \leq 1 \dots (3)$$

With time dependent G and Λ , the field equations of Einstein is specified as

$$R_{ij} - \frac{1}{2} R g_{ij} = \Lambda(t) g_{ij} - T_{ij} (8\pi G(t)) \dots (4)$$

In co-moving co-ordinate system, by using equation of metric (1) and equation of energy - momentum tensor (2),

Then (4) the field equation gives as

$$\frac{2B^{**}}{B} + \left(\frac{B^*}{B}\right)^2 = -8\pi G\rho + \Lambda \quad \dots (5)$$

$$\frac{A^{**}}{A} + \frac{B^{**}}{B} + \frac{A^*B^*}{AB} = -8\pi G\rho + \Lambda \quad \dots (6)$$

$$\frac{2A^*B^*}{AB} + \left(\frac{B^*}{B}\right)^2 = 8\pi G\rho + \Lambda \quad \dots (7)$$

" * " ,denotes ordinary differentiation,with respect to the cosmic time t.

Einstein tensor's equation, with respect to disappearing of divergence is given as

$$8\pi G \left[\rho^* + (\rho + p) \left[\frac{A^*}{A} + \frac{2B^*}{B} \right] \right] + 8\pi G^*\rho + \Lambda^* = 0 \quad \dots (8)$$

The equation of energy conservation gives($T_{i,j}^j = 0$)

$$\rho^* + (\rho + p) \left(\frac{A^*}{A} + \frac{2B^*}{B} \right) = 0 \quad \dots (9)$$

By substituting the value of equation (9) in the equation (8), we get G and Λ coupled field specified by

$$8\pi G^*\rho + \Lambda^* = 0 \quad \dots (10)$$

The equation (10) representing Λ is a constant when G is a constant.

By substituting the value of equation (3) in(9) equation, also by integrating,

$$\rho = \frac{k}{R^3(\omega+1)} \quad \dots(11)$$

here k signifies as the constant of integration that is $k > 0$

Volume of this model is specified by

$$V = R^3 = ABC \quad \dots (12)$$

We define Average scale factor R is $[ABC]^{1/3}$ of locally rotationally symmetric Bianchi type- I universe.

From the equations (5), (6) and (7)

$$\frac{A^*}{A} - \frac{B^*}{B} = \frac{k_1}{R^3} \quad \dots (13)$$

The Constant of integration is denoted by k_1 . The Hubble-parameter is denoted by 'H', and deceleration parameter is denoted by q , σ is shear and θ is the volume expansion.

$$\theta = 3H = \frac{3R^*}{R},$$

$$\sigma = \frac{K}{\sqrt{3}R^3},$$

$$q = -1 - \frac{H^*}{H^2} = -\frac{RR^{**}}{[R^*]^2}$$

In terms of H , σ and q , the equations (5) to (7) and also equation (9) can be specified by

$$(2q - 1)H^2 - \sigma^2 = 8\pi pG - \Lambda, \quad \dots (14)$$

$$3H^2 - \sigma^2 = 8\pi\rho G + \Lambda, \quad \dots (15)$$

$$\rho^* + 3(\rho + p)\frac{R^*}{R} = 0 \quad \dots (16)$$

According to Overduin, & Cooperstock, [40]

$$\rho_c = \frac{3H^2}{8\pi G} \quad \text{consider as critical density} \quad \dots (17)$$

$$\rho_v = \frac{\Lambda}{8\pi G} \quad \text{specifies as vacuum density} \quad \dots (18)$$

$$\Omega = \frac{\rho}{\rho_c} = \frac{8\pi G\rho}{3H^2} \quad \text{consider as density parameter} \quad \dots (19)$$

By substituting these values in the equation (15), we acquire

$$\frac{3\sigma^2}{\theta^2} = 1 - \frac{(24\pi G\rho)}{H^2} - \frac{3\Lambda}{\theta^2}$$

Signifies that for $\Lambda \geq 0$

$$0 \leq \frac{\sigma^2}{\theta^2} \leq \frac{1}{3},$$

$$0 \leq \frac{8\pi G\rho}{\theta^2} \leq \frac{1}{3}$$

Therefore the existence of negative lambda gives more for anisotropy whereas positive lambda lowers the upper limit of anisotropy.

From (14), and (15), we have

$$\begin{aligned} \frac{d\theta}{dt} &= -12\pi pG - \frac{3}{2}\sigma^2 + \frac{3}{2}\Lambda - \frac{\theta^2}{2} \\ &= -12\pi G(\rho + p) - 3\sigma^2 \end{aligned}$$

That indicates, during the evolution of time, the rate of volume extension reduces and therefore the existence of positive lambda implies that the universe is slowing down its rate of decay whereas a negative lambda would support it.

$$\sigma^* = -\frac{3\sigma R^*}{R}$$

It indicates that in the growing universe, the value of σ reduces and for infinitely value of R , the value of σ is insignificant.

III. SOLUTION OF THE FIELD EQUATIONS

Five equations in six unknowns (A, B, ρ, p, G and Λ) are provided by the system of equations (3), (5) –(7) and (10). For completely solving the system, an extra equation is required. Number of authors recognized the decline of lambda. Lambda approaches to $(-2)^{th}$ power of a which is denoted as the scale factor in the metric of Roberston –Walker, examined by Chen and Wu [9]. Lambda approaches to $(-3)^{th}$ power of a which is represented as the scale factor, examined by Pavon, D. [2], Holye, F. et al. [3], Olson, T.S. et al. [4], Maia, M.D. et al. [6], Silveria, V. et. al. [7, 8], Torres and L.F. B. et al. [9] observed the value of lambda approaches to $(-m)^{th}$ power of a which is indicated as scale factor where value of m is constant.

$$\Lambda = \frac{a}{R^m}, \text{ signifies a decaying vacuum energy density.} \quad \dots (20)$$

By substituting the equations (11) and (20) in the equation (10), then

$$G = \left[\frac{a}{8\pi k} \right] \left[\frac{R^{3\omega+3-m}}{3\omega+3-mk} \right] \quad \dots (21)$$

By using the equations (13), (18) and (19), we acquire

$$\frac{R^{**}}{R} + 2 \left[\frac{R^*}{R} \right]^2 - \frac{am(1-w)}{2(3\omega+3-m)R^m} - \frac{a}{R^m} = 0 \quad \dots (22)$$

3.1 When the value $\omega = 0$ then Vacuum solution

When the value $\omega = 0$,

The value of the equation (22) turn into

$$\frac{R^{**}}{R} + 2 \left(\frac{R^*}{R} \right)^2 - \frac{a(m-6)}{2(m-3)R^m} - \frac{1}{R^m} = 0 \quad \dots (23)$$

To acquire the time progression of Hubble parameter, integrate the equation (23)

$$\frac{R^*}{R} = H = \sqrt{\frac{a}{3-m}} \left[\frac{m}{2} \sqrt{\left(\frac{a}{3-m} \right)} t + t_0 \right]^{-1} \quad \dots (24)$$

here t_0 is a constant,

To acquire the value of scale factor, using the equation (24)

$$R = \left(\frac{m}{2} \sqrt{\left(\frac{a}{3-m} \right)} t + t_0 \right)^{2/m} \quad \dots (25)$$

By substituting the equation (25) into the equation (13), the equation of metric (1) exist as

$$ds^2 = dt^2 + \left(\frac{m}{2} \sqrt{\left(\frac{a}{3-m} \right)} t + t_0 \right)^{\frac{4}{m}} \times$$

$$\left[-m_1^2 \exp 2 \left\{ \frac{8k_1}{3} \left[\sqrt{\frac{3-m}{a}} \right] \frac{1}{m-6} \left(\frac{m}{2} \sqrt{\left(\frac{a}{3-m} \right) t} + t_0 \right)^{\frac{m-6}{m}} \right\} dx^2 - \right. \\ \left. m_2^2 \exp 2 \left\{ -4k_1 \left[\sqrt{\frac{3-m}{a}} \right] \frac{1}{m-6} \left(\frac{m}{2} \sqrt{\left(\frac{a}{3-m} \right) t} + t_0 \right)^{\frac{m-6}{m}} \right\} dy^2 + dz^2 \right] \dots (26)$$

Here m_1 and m_2 signifies as constants.

From the above model (26), ρ is denoted as matter density, p is denoted as pressure and G signifies as gravitational constant, Λ is denoted as cosmological constant and the spatial volume is denoted as V are specified as

$$V = \left(\frac{m}{2} \sqrt{\left(\frac{a}{3-m} \right) t} + t_0 \right)^{6/m}, \dots (27)$$

$$\rho = \frac{k}{\left[\frac{m}{2} \sqrt{\left(\frac{a}{3-m} \right) t} + t_0 \right]^{\frac{6}{m}}}, \dots (28)$$

$$p = 0, \dots (29)$$

$$G = \frac{am}{8\pi k (3-m)} \left[\frac{m}{2} \sqrt{\left(\frac{a}{3-m} \right) t} + t_0 \right]^{\frac{2}{m} (3-m)} \dots (30)$$

$$\Lambda = a \left[\frac{m}{2} \sqrt{\left(\frac{a}{3-m} \right) t} + t_0 \right]^{-2} \dots (31)$$

θ is denoted as expansion scalar, σ is signifies by shear

$$\theta = 3 \sqrt{\frac{a}{3-m}} \left[\frac{m}{2} \sqrt{\left(\frac{a}{3-m} \right) t} + t_0 \right]^{-1} \dots (32)$$

$$\sigma = \frac{k}{\sqrt{3}} \left[\frac{m}{2} \sqrt{\left(\frac{a}{3-m} \right) t} + t_0 \right]^{-6/m} \dots (33)$$

$$\Omega = \frac{\rho}{\rho_c} = \frac{m}{3} \text{ is the density parameter} \dots (34)$$

For acquired model, value of deceleration parameter q specifies as

$$q = \frac{m}{2} - 1 \dots (35)$$

ρ_v is denoted as the vacuum energy, ρ_c is denoted as density critical density are

$$\rho_v = \frac{k (3-m)}{m} \left[\frac{m}{2} \sqrt{\left(\frac{a}{3-m} \right) t} + t_0 \right]^{-6/m} \dots (36)$$

$$\rho_c = \frac{3k}{m} \left[\frac{m}{2} \sqrt{\left(\frac{a}{3-M}\right)} t + t_0 \right]^{-6/m} \quad \dots (37)$$

From the given model (26), it has been noticed that for $0 < m < 3$, If $t = t'$ then value of the spatial volume $V = 0$ and

also the value of $t' = \frac{-t_0}{\frac{m}{2} \sqrt{\frac{a}{3-m}}}$ and the value of θ which is denoted as expansion scalar is

infinite, that displays that our space begins to grow at $t=t'$ with zero volume and an infinite rate of development. At primary era, point type singularity is displayed by the space time. When there was initial singularity, at energy density the shear scalar deviates. When rises the value of scale factors and spatial volume rises, whereas expansion scalar declines. With increasing time, rate of growth slows down. And $\rho, \sigma, \rho_v, \rho_c, \Lambda$ reduces as t rises. Moreover the value of $\rho, \sigma, \rho_c, \rho_v$, and Λ tend to zero, as $t \rightarrow \infty$ then value of scale factors and volume become infinite.

Since huge time t , the model would basically provide a void universe. When rises, the value of $G(t)$ also rises and at $t=t'$ value of the gravitational constant $G(t)$ is zero.

Abdel – Rahman [13], Abdussattar [17], Chow [24], Levitt [25] and Milne [26] examined the sectional list of cosmological models wherever the gravitational constant is growing.

The proportion $\frac{\sigma}{\theta}$ approaches to zero as t approaches to ∞ offered $m < 3$.

Above model tends to isotropy for the hefty value of t . thus it signifies shearing, non-rotating and growing model for the cosmos with a big bang start approaching to isotropy at late times.

This has examined while $q < 0$, for $0 < m < 2$,

$$q = 0, \text{ for } m = 2,$$

$$q > 0; \text{ for } 2 < m < 3$$

IV. CONCLUSION

- In this manuscript we have acquired the cosmological models that shows the universe starts to accelerate and increases exponentially with infinite time t .
- Model (26) concludes dynamically anisotropic expansion to the universe which allows for symmetrical Bianchi metrics.
- Hence the space starts through a decelerating extension deviates and the growth deviates through decelerating phase to accelerating one.

- With Supernovas la astronomical observations, this cosmological scenario is in accordance whichoffers a unified description of the growth of the cosmos.

V. FUTURE SCOPE

With the advancement in astrophysical studies and latest technologies, there is lot more to be observed, analyzed and learned in our never-ending universe. According to multiverse theory, there can be possibilities of many alternative universes with different timelines, different laws of physics, never-ending possibilities, and new concepts.

REFERENCES

- [1] Kramer, D., &Schmutzer, E., "In Exact solutions of Einstein's field equations, (New York: Cambridge University Press), 1980.
- [2] Pavon, D., "Nonequilibrium Auctuations in cosmic vacuum decay", Phys. Rev. D, Vol.43, No.2,pp.375-378, 1991.
- [3] Hoyle, F., Burbidge, G., & Narlikar, J. V., "On the Hubble constant and the cosmological constant", Monthly Notices of the Royal Astronomical Society, Vol.286, No. 1pp.173-182,1997.
- [4] Olson, T. S., & Jordan, T. F., "Ages of the Universe for decreasing cosmological constants", Phys. Rev. D, Vol. 35, No. 10, pp.3258-3260,1987.
- [5] Maia, M. D., & Silva, G. S., "Geometrical constraints on the cosmological constant M", Phys. Rev. D, Vol.50, No.12,pp.7233-7238, 1994.
- [6] Silveira, V., &Waga, I., "Decaying Λ cosmologies and power spectrum", Phys. Rev. D, Vol.50, No.8, pp.4890-4894,1994
- [7] Silveira, V., &Waga, I. 1997, "Cosmological properties of a class of [Formula presented] decaying cosmologies", Phys. Rev. D, Vol.56, No. 8,pp.4625-4632,1997.
- [8]Bloomfield Torres, L. F., &Waga, I., "Decaying Λ cosmologies and statistical properties of gravitational lenses", Monthly Notices of the Royal Astronomical Society, Vol.279, No.3, pp.712-726, 1996.
- [9] Chen, W., & Wu, Y.-S., "Implications of a cosmological constant varying", Phys. Rev. D, Vol. 41,pp. 695-698,1990.
- [10] Carvalho, J. C., Lima, J. A. S., &Waga, I. , "Cosmological consequences of a time-dependent Lambda term", Phys. Rev. D, Vo.46, No. 6,pp.2404-2407,1992.
- [11] DIRAC, P. A. M. "The Cosmological Constants", Nature, Vol.158, pp.323-323, 1937.
- [12] Lau, Y.-K. 1985, "The Large Number Hypothesis and Einstein's Theory of Gravitation", Australian Journal of Physics, Vol.38, No. 4,pp.547-553, 1985.
- [13] Abdel-Rahman, A.-M. M., "A critical density cosmological model with varying gravitational and cosmological "constants", "General Relativity and Gravitation, Vol.22, no.6,pp. 655-663, 1990.
- [14] Berman, M. S., &Som, M. M." Static universe in a modified brans-dicke cosmology", International Journal of Theoretical Physics, Vol. 29, No.6, pp. 567-570, 1990.
- [15] Sister'o, R. F." Cosmology with G and A Coupling Scalars", General Relativity and Gravitation, Vol.23, No.11, pp.1265-1278, 1991.

- [16] Kalligas, D., Wesson, P., &Everitt, C. W. F.,” Flat FRW Models with Variable G and A”,General Relativity and Gravitation,Vol. 24, No.4, pp.351-357, 1992.
- [17] Abdussattar, &Vishwakarma, R. G.,” FRW models with variable G and Λ ”, Classical and Quantum Gravity, Vol. 14, No.4, pp.945-953, 1997.
- [18]Vishwakarma, R. G.,” Λ Decaying As Energy Density: A Study Of Angular Size - Redshift Relation”, Classical and Quantum Gravity, Vol.17, No.18, pp.3833-3842, 2000.
- [19] Vishwakarma, R. G., “A model to explain varying Λ , G and σ^2 simultaneously“, General Relativity and Gravitation, Vol.37, No.7, pp.1305-1311, 2005.
- [20] Pradhan, A., &Otarod, S.,” A new class of bulk viscous universe with time dependent deceleration parameter and Λ -term”,Astrophysics and Space Science, Vol. 311, No.4, pp.413-421, 2007.
- [21] Singh, C. P., Kumar, S., & Pradhan, A., “Early viscous universe with variable gravitational and cosmological ‘constants’C ”, Classical and Quantum Gravity, Vol.24, No.2,pp.455-474,2007.
- [22] Singh, J. P., & Tiwari, R. K.,” Perfect fluid Bianchi type I cosmological models with time varying G and Λ ”Pramana, Vol.70,No.4 pp.565-574,2008.
- [23] Borges, H. A., &Carneiro, S.,”Friedmann cosmology with decaying vacuum density”, General Relativity and Gravitation, Vol.37, No. 8, pp.1385-1394, 2005.
- [24] Chow, T. L.,” The variability of the gravitational constant”,NuovoCimentoLettere, Vol.31, No.4, pp.119-120, 1981.
- [25] Levitt, L. S.,” The gravitational constant at time zero”,NuovoCimentoLettere, Vol. 29, No.1, pp.-24, 1980.
- [26] Milne, E. A., “Relativity and cosmology”,Relativity, Gravitation and world structure ,Nature, 1935.
- [27] Knop, R. A., et al.,”New Constraints On High-Redshift Supernovae Observed With The Hubble Space Telescope”,The Astrophysical Journal,Vol. 598, pp.102-137,2003.
- [28] Riess, A. G., et al.,“The High-Z Supernova Search: Measuring Cosmic Deceleration And Global Curvature Of The Universe Using Type Ia Supernovae1”, The Astrophysical Journal, Vol 46,No.1, pp. 46-63, 1998.
- [29] Riess, A. G., et al.,” Type IaSupernova Discoveries At From The Hubble Space Telescope: Evidence For Past Deceleration And Constraints On Dark Energy Evolution”,The Astrophysical Journal, Vol.607, pp.665-687, 2004.
- [30]Spergel, D. N., et al.” Three-Year Wilkinson Microwave Anisotropy Probe (Wmap) Observations: Implications for Cosmology”,The Astrophysical Journal Supplement Series, Vol. 170, pp. 377-408, 2007.
- [31] Tegmark, M., et al.,” Cosmological Parameters From SDSS And WMAP”,Phys. Rev. D, Vol. 69, No.10,pp.1-26,2004.
- [32] Kalligas, D., Wesson, P., &Everitt, C. W. F.,” Flat FRW Models With Variable G And A”,General Relativity And Gravitation,Vol. 24, No.4, pp.351-357,1992.
- [33] Berman, M. S.,” Static Universe In A Modified Brans-DickeCosmology”, International Journal of Theoretical Physics, Vol.29, No.6, pp.567-570, 1990.
- [34] Berman, M. S., &Som, M. M., “Brans-Dicke models with time-dependent cosmological term”, International Journal of Theoretical Physics, Vol.29, No.12, pp.1411-1414, 1990.
- [35]Berman, M. S., Som, M. M., & de Mello Gomide, F. ,”Brans-Dicke static universes”, General Relativity and Gravitation, Vol.21, No.3,pp.287-292,1989.
- [36]Bertolami, O.,” Time-dependent cosmological term”,NuovoCimento B Series, Vol.93, No.1,pp.36-42, 1986.

- [37] Bertolami, O. "Brans-Dicke Cosmology with a Scalar Field Dependent Cosmological Term", *Fortschritte der Physik*, Vol.34, No.12, pp. 829-833, 1986.
- [38] Perlmutter, S., et al., "Discovery of a supernova explosion at half the age of the Universe", *Nature*, Vol.39, No.1, pp.51, 1998.
- [39] Weinberg, S. ed. Steven Weinberg, "Principles and Applications of the General Theory of Relativity", in *Gravitation and Cosmology*, pp.1-685, 1972.
- [40] Overduin, J. M., & Cooperstock, F. I., "Evolution of the scale factor with a variable cosmological term", *Phys. Rev. D*, Vol.58, No.4, pp.1-23, 1998.