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Using Calculus of Variations to Find the Path of a Rocket in Minimum Time

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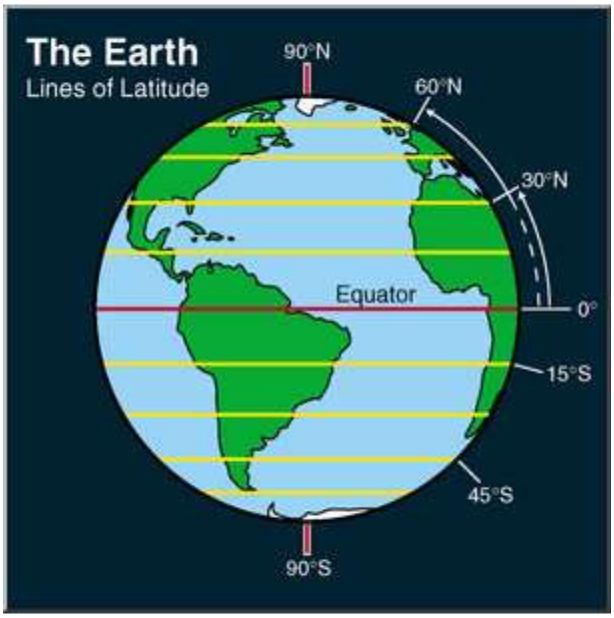
ABSTRACT
This paper attempts to find the minimum time to attain the escape velocity for a single staged rocket if the path of the rocket is along exponential curves. From this calculation, we determine if an ideal path of a rocket can reach escape velocity in the least time.

In this paper, r (radius of lattitute) has been considered according to the position of the rocket.

Keywords: Euler-Lagrange equation, Escape Velocity, Universal Gravitational Constant,G = $6.673\times10^{-11}\,Nm^2/kg^2$ *r is defined as the radius of latitude as per position of the rocket (and not the radius of the earth).*



INTRODUCTION
Escape velocity is the minimum speed needed For an object to escape from the gravitational Influence of a massive body. The escape velocity From the Earth is approximately 11.186 km/s at the surface (when not considering air resistance and its variation with height).

Escape velocity is defined by

$$V = \sqrt{2GM/r}$$

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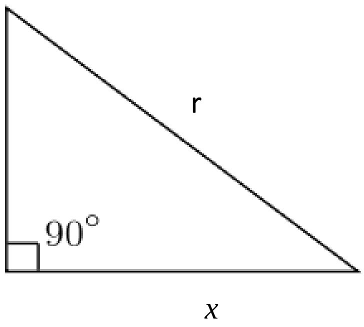
Vol-22-Issue-40-December-2019

$$\frac{ds}{dt} = (\sqrt{2GM})\, r^{-1/2}$$



$$\frac{ds}{dt} = \sqrt{2GM}\,(x^2 + y^2)^{-\frac{1}{4}}$$

(Since $ds = \sqrt{(1 + (\frac{dy}{dx})^2}\, dx$)

$$dt = \frac{1}{\sqrt{2GM}}(x^2 + y^2)^{\frac{1}{4}} \sqrt{1 + y'^2}\; dx$$

The minimum time T required for the Escape Velocity

$$T = \int_{t_1}^{t_2} dt = \int_{x_1}^{x_2} \frac{1}{\sqrt{2GM}}(x^2 + y^2)^{\frac{1}{4}} \sqrt{1 + y'^2}\; dx$$

By Euler's theorem

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$$

$$\frac{1}{4}\frac{\sqrt{1 + y'^2}}{\sqrt{2GM}}(x^2 + y^2)^{-\frac{3}{4}} 2y - \frac{d}{dx}\left(\frac{1}{2\sqrt{2GM}}(x^2 + y^2)^{\frac{1}{4}} (1 + y'^2)^{-\frac{1}{2}})2y'\right) = 0$$

$$d\left(\frac{1}{2\sqrt{2GM}}(x^2 + y^2)^{\frac{1}{4}} (1 + y'^2)^{-\frac{1}{2}})2y'\right)$$

$$= \frac{1}{4}\frac{\sqrt{1 + y'^2}}{\sqrt{2GM}}(x^2 + y^2)^{-\frac{3}{4}} 2y\, dx$$

$$(x^2 + y^2)^{1/4} \frac{y'}{\sqrt{1 + y'^2}}$$

$$= \frac{1}{2}\sqrt{1 + y'^2}\; y \int (x^2 + y^2)^{-\frac{3}{4}}\, dx$$

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$$= \frac{1}{2}\sqrt{1+y'^2}\; y\int (y^2\tan^2\theta + y^2)^{-\frac{3}{4}}\, y\sec\theta\tan\theta\, d\theta$$

$$= \frac{1}{2}\sqrt{1+y'^2}\; y^2 \int (y^2)^{-\frac{3}{4}} (\sec^2\theta)^{-3/4}\, \sec\theta\tan\theta\, d\theta$$

$$= \frac{1}{2}\sqrt{1+y'^2}\; (y^2)^{-\frac{1}{4}} \int K^{-\frac{3}{2}} dK$$

$$= \frac{1}{2}\sqrt{1+y'^2}\; (y^2)^{-\frac{1}{4}} \frac{K^{-\frac{1}{2}}}{-\frac{1}{2}} + C$$

$$= -\sqrt{1+y'^2}\; (y^2)^{-\frac{1}{4}} (\sec\theta)^{-\frac{1}{2}} + C$$

$$= -2\sqrt{1+y'^2}\; (y^2)^{-\frac{1}{4}} \left(\sqrt{\frac{x^2+y^2}{y^2}}\right)^{-\frac{1}{2}} + C$$

$$(x^2+y^2)^{1/4} \frac{y'}{\sqrt{1+y'^2}}$$

$$= -\sqrt{1+y'^2}\; (x^2+y^2)^{-\frac{1}{4}}$$

$(Since\ C\ is\ arbitrary)$

$$(x^2+y^2)^{\frac{1}{2}}\, y' + (1+y'^2) = 0$$

Put $y' = m$ *and* $(x^2+y^2)^{\frac{1}{2}} = L$

Hence the equation becomes

$m^2 + Lm + 1 = 0$

So $m = \dfrac{-L \pm \sqrt{(L^2-4)}}{2}$

When $x \to 0$ *we have* $L \to y$

Hence $m = ay$

But $m = \dfrac{dy}{dx}$

$$\frac{dy}{dx} = ay \qquad \frac{dy}{y} = a\,dx$$

On integration $y = b e^{ax}$

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CONCLUSION

Therefore, to attain the escape velocity by a rocket in the minimum time path should be along exponential curve.

REFERENCES

- Fundamentals of Physics – David Halliday, Robert Resnick, Jearl Walker – 9th Edition, Wiley, 2010
- http://math.arizona.edu/~jaytaylor/files/york/CalculusofVariations.pdf
- Classical Mechanics - H Goldstein, C Poole & J Safko - 3rd Ed., Pearson, 2002

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