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MATHEMATICAL MOEL ON CRIMINAL ACTIVITY MOVEMENT IN SOCIAL

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Abstract-

The problem of Criminal activities in our societies remains a major challenge. It is very difficult to eradicate Criminal activities from the societies. In recent years many researchers highlight applied mathematics as important literature resources that may help us better understand criminal activity. In this paper we are framing mathematical model based on movement of criminal activity and formation of crime hotspots. We also propose adding repeated victimization leaving aside the term 'risk associated with cost for criminal activities' in ourmodel.

1.Introduction:.

Criminal Activity depends upon the place and time. The Spatial-temporal behavior of the criminal activity can be studied in many categories. Criminal activity generally does not occur in day-light except metropolitan cities. However, in the remote places, night and evening time is preferred for stealing and other crimes. Highway robbery often takes place at lonely and non-congested road at midnight. The peak hour of marketing, vehicle or bike theft is much more common than in normal time at parking places. Criminal activity – such as burglary, larceny *etc*. are committed during day time in the absence of capable owners and during evening time with the help of darkness [12,13] . Criminal activity related to sex is committed in the lonely place at late night. Murder and sexual crimes take place at evening or at night in bars and restaurants of metropolitan cities. The departure time of trains from stations is the boon time for pick-pockets to commit crimes like snatching *etc*. When a mob becomes uncontrollable, criminal finds an opportunity to indulge themselves in loot and damage of the national and privategoods.

Mathematical models using applications from different subjects related to the field of criminology with their own methods. Several mathematical models was developed based on criminology theory. As for examples Economic crime models were presented by G.S.Becker [1], and Isaac Ehrlich [2,3,4], social crime models were proposed by Glaeser *et al* [5,6]. There is a plenty of publications available in the other field of crime. Out of them, we may mentioned the relation between education and crime [7], relation between crime and unemployment[8]. Cambel *et al* [9] treat criminality as an problems base on epidemics with the help of differential equations. Another mathematical model of crime was introduced

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[10] using population dynamics.

We are interested to focus our attention on the dynamics of criminal activity was recently presented in [11]. The author derived a dynamics of criminal activity and formation of crime hotspots. In the present paper, dynamics of criminal activity is formed leaving the risk aversion term and adding the effect of repeated victimization [12].

2.Formulatiuon of the model:There is a functional relation between the number of criminal activity and place(spatial) and time (temporal). If we denote the number of criminal activity by A and place x at time t .This relation can be written as A=A(x,t) .Further we let T be the tendency (or propensity in [13] of criminal to commit crime at the place x at time t.This relation can be expressed as T=T(x,t).With the hypothesis ,that every potential criminal have a monotonic increasing tendency to act crime .It is remarkable that according to the routine activity theory of every day life ,it is termed that the criminal activity is possible only for the potential criminal ,in presence of suitable targets and in the weak strength of security [12,13].

Now we consider a city which is defined as an open set X .We also consider that the a number of activity A(x,t) at place x and at time t is represented by the function of tendency of criminal activity T(x,t).This can be written as A(x,t) = F[T(x ,t)] this implies that A(x,t) = F(T) (1)Where T=T(x,t) and F is the acting out function [3] ,which can be written as

$$F(T) = \begin{cases} 0, & T \leq 0 \\ 1 - e^{-\gamma T}, & T > 0 \end{cases}$$

For $\lambda > 0$ $is\ given\ \ constant\ and\ F`(0) = \gamma$ which measures the strength of a positive tendency T(x,t).In particular γ can be taken as a parameter that varies in spatial location. The spatial-temporal field T(x,t) represents such a condition that when T > 0,criminal activity is successful and if T < 0,the criminal have to face high risk to commit the criminal activity at x on time t .For the simplicity we can consider $T \to \infty$. So that $(1 - e^{-\gamma t}) \to 1$ so that acting out function F(T) can be considered as a step function-

$$F(T) = \begin{cases} 0, & T \leq 0 \\ 1, & T > 0 \end{cases}$$

Thus this function has binary nature of decision to work or not to work the criminal activity.In this cased strength γ of tendency T(x,t) of the criminal becomes very strong

3.Properties of the acting out function F

(i)$\mathrm{F[T(x.t)]} = 0, \mathrm{for T(x,t)} \leq 0, \mathrm{F[T(x.t)]} > 0$

(ii) F is a increasing function of T $> 0\ with\ F \to 1\ when\ T \to \infty$

The movement of criminal activity can be written as according to Berestycki and Nadal`s -

$$\frac{\partial \mathrm{T}}{\partial \mathrm{t}} = -\mathrm{T(x,t)} + \mathrm{P(x,t)} - \mathrm{C(x,t)A_m(x,t)} \qquad (2)$$

$$\mu_{\mathrm{A}} \frac{\partial \mathrm{A_m}}{\partial \mathrm{t}} = \mathrm{A(x,t)} - \mathrm{A_m(x,t)} \qquad (3)$$

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$$\mu_A \frac{\partial A_m}{\partial t} = F[T(x,t)] - A_m(x,t)$$

Where $A_m =$ the moving mean of past crimanal activity history of rate of crime

$C(x,t) =$ cost of commiting a crime or cost for punishment by police/army

$P(x,t) =$ local propensity to act of a crime

$C(x,t)A_m(x,t) =$ risk associated with criminal activity

$\mu_A =$ relativetime scale for the criminalactivity $A_m(x,t)$ and $\mu_A > 1$.

We assume that the environment is uniform in the propensity to act as the local expected payoffi i. e

$P(x,t) \approx P(x)$Then the equation(3a)express the relation of $T(x,t)$ towards $P(x)$

which is the average value of $P(x,t)$ over the spatial domain.

V Time Independent of P and C in spatial location

We assume that P(x,t) and C (x,t) are independent of time such that P(x,t)=P(x) and C(x,t)=C(x) .

Then the equation (30 can be written in spatial location as

$$\frac{\partial T(x,t)}{\partial t} = -T(x,t) + P(x) - C(x)A_m(x,t) \qquad (4)$$

$$\mu_A \frac{\partial A_m(x,t)}{\partial t} = FT[(x,t)] - A_m(x,t) \qquad (5)$$

If we dropped the spatial term x, then two coupled equations (4) & (5) for the scalars C >0 and P can be written as

$$\frac{\partial T(t)}{\partial t} = -T(t) + P - CA_m(t) \qquad (6)$$

$$\mu_A \frac{\partial A_m(t)}{\partial t} = F[T(t)] - A_m(t) \qquad (7)$$

4.Equilibrium Points of the movement of criminal activities:

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We let (T^0, A_m^0) be the equilibrium point of the above system then the equilibrium points are obtained from

equation (5) ,(6) $\frac{\partial T(t)}{\partial t}$ and $\mu_A \frac{\partial A_m(t)}{\partial t}$ to zero when $T(t) \to T^0$ $and\ A_m(t) \to A_m^0$ .So we have

$$\frac{\partial \mathrm{T(t)}}{\partial \mathrm{t}} = 0 \text{ implies that } -\mathrm{T^0} + \mathrm{P} - \mathrm{CA_m^0} = 0 \text{ and } \mu_\mathrm{A} \frac{\partial \mathrm{A_m(t)}}{\partial \mathrm{t}} = 0 \text{ , provides } \mathrm{F(T^0)} - \mathrm{A_m^0}$$

$\mathrm{T^0} = \mathrm{P} - \mathrm{CA_m^0}$ and$\mathrm{A_m^0} = \mathrm{F(T^0)}$ (8)

$\mathrm{T^0} = \mathrm{P} - \mathrm{CF(T^0)}$

$\mathrm{A_m^0} = \mathrm{F(T^0)}$ (9)

By the property of (2) and (3) of acting out function F[T(x,t)] ,we have F(T)=0 for $T \leq 0\ and$

$F(T) > 0\ for\ T > 0$. F is a increasing function on $[0, \infty]$ $when\ T > 0\ with\ F \to 0\ when\ T \to \infty$

Satisfying the $\mathrm{F(0^+)} = 0$ when γ is defined by $\gamma = \mathrm{F`(0^+)}and\mathrm{F(T)} = 1 - \mathrm{e^{-\gamma T}}$

using these properties of F ,we find the there exist a unique solution of the system (5) and (6) for all of

values of (P,C). When P < 0 (T <) we get $\mathrm{F(T^0)} = 0$

$\mathrm{T^0} = \mathrm{P}$ (10)

$A_m^0 = 0$ (11)

So the equilibrium point $(\mathrm{T^0}, \mathrm{A_m^0})$ is given by (P, 0) for $\mathrm{P} > 0$, $\mathrm{T^0} > 0$ $is\ the\ unique$ intersection of the graph $\emptyset(\mathrm{T}) = \mathrm{T}and\emptyset(\mathrm{T}) = \mathrm{P} - \mathrm{CF(T)}for\mathrm{T} \geq 0$

This gaurantee (T) becomes high For greater positive (P) and consequently the rate of the criminal activity A^m become very large number.

5. Analysis of the model:

The stability is determined by linearization of equation (5) and (6) near the equilibrium point

$(T^0, A_m^0) = (P, 0)$ as in the equation (8 and (9) .Now the whole system can be written as

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$$\frac{\partial T(t)}{\partial t} = -T(t) + P - CA_m(\mathrm{t}) = \mathrm{f}(\mathrm{T}, A_m)$$

$$\frac{\partial A_m(t)}{\partial t} = \frac{1}{\mu_A} F[T(t)] - \frac{1}{\mu_A} A_m(t) = g(T, A^m)$$

Jacobian matrix of the right side of the system at the equilibrium point $(T^0, A_m^0) = (P, 0)$ *is given by*

$$J_{(T^0, A_m^0)} = \begin{bmatrix} \frac{\partial f}{\partial T} & \frac{\partial f}{\partial A_m} \\ \frac{\partial g}{\partial T} & \frac{\partial g}{\partial A_m} \end{bmatrix}_{(T^0, A_m^0)} = \begin{bmatrix} -1 & -C \\ \frac{1}{\mu_A} F`(T) & -\frac{1}{\mu_A} \end{bmatrix}_{(P,0)}$$

$$= \begin{bmatrix} -1 & -C \\ \frac{1}{\mu_A} F`(P) & -\frac{1}{\mu_A} \end{bmatrix} = \begin{bmatrix} -1 & -C \\ 0 & -\frac{1}{\mu_A} \end{bmatrix} (12)$$

Near the equilibrium point , $P < 0$, $F`(P) = F`(T^0) = 0$ .The Jacobian matrix (10) is a triangular matrix. Hence the eigen values of the matrix (10) are

$$\lambda_1 = -1 \; and \; \lambda_2 = -\frac{1}{\mu_A} \qquad (13)$$

Now it is clear that two eigen values of the Jacobian matrix J are negative .Hence the whole system (5) is stable .

Again Jacobian matrix , near the equilibrium point (T^0, A_m^0) is $J_{(T^0, A_m^0)} = \begin{bmatrix} -1 & -C \\ \frac{1}{\mu_A} F`(T) & -\frac{1}{\mu_A} \end{bmatrix}_{(T^0, A_m^0)}$

when P>0 is $J_{(T^0, A_m^0)} = \begin{bmatrix} -1 & -C \\ \frac{1}{\mu_A} F`(T^0) & -\frac{1}{\mu_A} \end{bmatrix}$ $as\ P > 0$ , $\gamma^0 = F`(T^0)$ $as\ \gamma = F`(0^+)$

$$J_{(T^0, A_m^0)} = \begin{bmatrix} -1 & -C \\ \frac{\gamma^0}{\mu_A} & -\frac{1}{\mu_A} \end{bmatrix} \; is \; 2 \times 2 \; matrix$$

$if\ \lambda$ be the eigen values then we have the characteristic equation $|J - \lambda I| = 0$

$$\begin{vmatrix} -1 - \lambda & -C \\ \frac{\gamma^0}{\mu_A} & \frac{-1}{\mu_A} - \lambda \end{vmatrix} = 0$$

$$(-1 - C)(-\frac{1}{\mu_A} - \lambda) + \frac{\gamma^0 C}{\mu_A} = 0$$

$$\mu_A \lambda^2 + (\mu_A + 1)\lambda + (1 + \gamma^0 C) = 0$$

$$\lambda = \left[-(\mu_A + 1) \pm \{(\mu_A - 1)^2 - 4\mu_A \gamma^0 C\}^{1/2}\right] / 2\mu_A$$

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We know that for the real roots the discriminant $(\mu_A - 1)^2 - 4\mu_A\gamma^0 C \geq 0$

$(\mu_A - 1)^2 \geq 4\mu_A\gamma^0 C$

$\frac{(\mu_A-1)^2}{4\mu_A} \geq \gamma^0 C$ (14)

This gives the two real eigen values as

$\lambda_{1,2} = \frac{-(\mu_A+1)\pm\{(\mu_A-1)^2-4\mu_A\gamma^0 C\}^{1/2}}{2\mu_A}$ (15)

From equation (13) and (14) ,it is clear that the real part of the eigen values of the Jacobian matrix at the equilibrium point for the systems (5) & (6) are negative .Thus the equilibrium point is always stable.

Now $(\mu_A - 1)^2 - 4\mu_A\gamma^0 C < 0$ implies that $\frac{(\mu_A-1)^2}{4\mu_A} < \gamma^0 C$ (15)

This gives two complex eigen values from (13) Let $\lambda_{3,4} = \frac{-(\mu_A+1)\pm\{(\mu_A-1)^2-4\mu_A\gamma^0 C\}^{1/2}}{2\mu_A}$ (16)

Since $\frac{(\mu_A-1)^2}{4\mu_A} < \gamma^0 C$,the limit of γ^0 is taken into account as large enough .So P > C ,if the function F(T) has a steep slope at T >0 as shown in the fig ,.So γ^0 is considered large enough. For this reason we have got complex eigen values and convergence towards the equilibrium point with damped oscillations.

6.The Modified model for criminal activity

In the previous model ,Berestycki and Nadal proposed for time evalution of T added a quantity local cost C(x,t) of criminal activity.They subtracted the product C (x,t)A(x,t) ,which represents the risk associated with criminal activity for a criminal due to differen factors or other environment) from the system .

In this presented model this risk factor is neglected for a potential criminal in the location where the deterrent factors are very weak. More ever repeated victimization effect $\rho(x,t)\, A_m(x,t)$ as given in equation (13) is added ,where ρ is the density of criminals.[15,16].

In the absence of risk aversion and in the presence of repeated victimization effect [16] the following system occurs as

$\frac{\partial T}{\partial t} = -T(x,t) + P(x,t) + \rho(x,t)A_m(\mathrm{x,t})$ (17)

$$\frac{\partial A_m}{\partial t} = \frac{1}{\mu_A}[F(T(x,t)) - A_m(x,t)]$$

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We restrict the analysis to the simplest case i.e with time free values of P and ρ and without risk of criminal act , the system takes the form

$\frac{\partial T(t)}{\partial t} = -T(t) + P + \rho(x,t)A_m(t)$ (18)

$\frac{\partial A_m}{\partial t} = \frac{1}{\mu_A}[F(T(x,t)) - A_m(t)]$ (19)

The equilibrium points of the above system are obtained as $T^0 = P$, $A_m^0 = 0$ and the Jacobian is given by

$$J = J_{(P,0)} = \begin{bmatrix} -1 & \rho \\ \frac{1}{\mu_A}F`(T) & -\frac{1}{\mu_A} \end{bmatrix}_{(P,0)} = \begin{bmatrix} -1 & \rho \\ 0 & -\frac{1}{\mu_A} \end{bmatrix}$$

So the eigen values are -1 and $-\frac{1}{\mu_A}$ and hence the system (18) is stable .Clearly the dynamics are the same as for the simplest case discussed in [13]

7.Analysis of Criminal Activity

We again consider the acting out functions

$$F(T) = \begin{cases} 0, & T \le 0 \\ 1 - e^{-\gamma T}, & T > 0 \end{cases}$$

When $\gamma \to \infty$, $F\ converges\ to\ the\ step\ function\ as$

$$F(T) = \begin{cases} 0, & T \le 0 \\ 1, & T > 0 \end{cases}$$

For the equilibrium point of (6) and (7) we estimate as below

$\gamma \to \infty$, $when\ P > C$, $T^0 : P - C$

And $A_{m:}^0 : 1 = \begin{bmatrix} QT^0 = P - CA_m^0 \\ A_m^0 = F(T^0) : 1 \end{bmatrix}$ hot spot when P > C

Thus when $\gamma \to \infty\ we\ have\ T^0 = P - C\ and\ A_m^0 = 1\ for\ P > C$ .In this case there is a criminal hotspot.

$$F(T) = \begin{cases} 0, & T \le 0 \\ 1 - e^{-\gamma T}, & T > 0 \end{cases}$$

$$\frac{\partial T(t)}{\partial t} = -T(t) + P - CA_m(t)$$

$$\mu_A \frac{\partial A_m(t)}{\partial t} = F[T(t)] - A_m(t)$$

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8.Conclusion

The acting out function of tendency to commit crime determines the criminal activity.The spatial locations are characterized by the local propensity to act is greater than cost for punishment then tendency of criminal activity is given by the gap between cost and local propensity to act and then the criminal activity ismaximalA_m^0 ： 1 , Then in this case there is a criminal hotspot .The movement of criminal activity is not affectedby the removal of risk associated with criminal

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