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Solution of KdV Equations of Order Three and Four Using Accelerated HPTM

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Abstract

We have proposed accelerated homotopy perturbation transformation method (HPTM) for the solution of KdV equations of order three and four. The proposed technique is a blend of "homotopy perturbation technique" and "Laplace transformation" and include a new form of He's polynomial for handling the nonlinear terms present in the equation. The result obtained from the proposed method shows the reliability, effiectiveness and applicability of this method for solving such complex nonlinear partial differential equations.

1. Introduction

KdV equation was presented by de Vries and Korteweg in 1895. The equation has numerous applications and is used to portray countless wonders of astrophysical and physical phenomena like wave phenomena in enharmonic crystals; the waves that occur in shallow water waves; ion-acoustic waves that occur in plasma; solitary waves etc. Solitons have very interesting properties as these are waves transmit with same shape and constant velocity and even after mutual collision these waves remain stable. KdV equation represents the mathematical model of the solitons. Wazwaz [9] has applied Adomian decomposition method (ADM) to obtain the one and two soliton solution of KdV equation. Kaya and Aassila [10] implemented ADM to obtain the homogeneous and non-homogenous KdV equations. Kaya and El-Sayed [11] applied ADM to find the solution of fifth order and generalized KdV equations and further, the condition of convergence of ADM is also discussed. Kaya [12] used ADM to obtain the generalized KdV equations of order three and four. Yildrim [13] applied homotopy perturbation method (HPM) to find the solution of modified KdV equation. Kazeminia et.al. [14] employed variational iteration method to solve the generalized KdV equation of order three and four. Sharma et.al [15] have applied Homotopy perturbation transformation (HPTM) to find the solution of fifth order KdV equation and in [16] the solution of some fractional nonlinear partial differential equation is obtained by using homotopy perturbation Sumudu transformation method. Further, Sharma et.al. [4] used HPTM for solution of coupled nonlinear PDE's and they have applied Accelerated homotopy perturbation transformation method, which is blend of HPM [1-2] and Laplace transformation. Khan [7-8] introduced HPTM by combining HPM with Laplace transformation for the solution of nonlinear equation using He's polynomial. Here, we have used a new form of He's polynomial introduced by Kalla [3] which accelerate the convergence of the HPTM.

2.Accelerated HPTM

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To elucidate the proposed technique, consider the following nonlinear equations.

$$\frac{\partial \phi}{\partial t} + L\phi(x,t) + N\phi(x,t) = \mathrm{G}(x,t) \text{ with condition } \phi(x,0) = k(x) \tag{1}$$

On applying Laplace transformation to eq. (1) , we have

$$\mathcal{L}\left\{\frac{\partial \phi}{\partial t} + L\phi(x,t) + N\phi(x,t)\right\} = \mathcal{L}\{\mathrm{G}(x,t)\} \tag{*}$$

Applying properties of Laplace transformation to eq.(*), we get

$$\bar{\phi}(x,s) = \frac{1}{s}\,\phi(x,0) + \frac{1}{s}\mathcal{L}\{\mathrm{G}(x,t) - \{L\phi(x,t) + N\phi(x,,t)\}\} \tag{2}$$

Further, operating inverse Laplace transformation to eq.(2) gives,

$$\phi(x,t) = \phi(x,0) + \mathcal{L}^{-1}\left\{\frac{1}{s}\,\mathcal{L}\{\mathrm{G}(x,t) - \{L\phi(x,t) + N\phi(x,t)\}\}\right\} \tag{3}$$

Now, appling homotpoy perturbation method, we have

$$0 = (1-p)\big(\phi(x,t) - \phi(x,0)\big) + p\left(\phi(x,t) - \mathcal{L}^{-1}\left\{\frac{1}{s}\,\mathcal{L}\{\mathrm{G}(x,t) - \{L\phi(x,t) + N\phi(x,t)\}\}\right\}\right)$$

where $p \in [0,1]$ is a parameter. Let

$$\phi(x,t) = \sum_{n=0}^{\infty} \phi_n p^n \tag{4}$$

and

$$N\phi(x,t) = \textstyle\sum_{n=0}^{\infty} \widetilde{H}_n p^n, \tag{5}$$

where $\widetilde{H}_n$ represents accelerated He's polynomial with

$$\widetilde{H}_n(\phi_0, \phi_1, \phi_2 \ldots, \phi_n) = N(S_n) - \textstyle\sum_{i=0}^{n-1} \widetilde{H}_i,\ n \geq 1 \tag{6}$$

Where $\widetilde{H}_0 = N(\phi(x_0))$, and $S_k = (\phi_0 + \phi_1 + \phi_2 + \cdots + \phi_k)$. Using (4), (5) and (6) the eq. (3) gives,

$$\textstyle\sum_{n=0}^{\infty} \phi_n p^n = \phi(x0) + p\mathcal{L}^{-1}\left\{\frac{1}{s}\,\mathcal{L}\left\{\Psi(x,t) - \left\{L\sum_{n=0}^{\infty} \phi_n p^n + \sum_{n=0}^{\infty} \widetilde{H}_n p^n\right\}\right\}\right\} \tag{7}$$

Now, comparing coefficients of the like power of p, we have

$$p^0 : \phi_0 = \phi(x,0)$$

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$$p^1: \phi_1 = \mathcal{L}^{-1}\left\{\frac{1}{s}\,\mathcal{L}\left\{\Psi(x,y,t) - \left\{L\phi_0 + \widetilde{H}_0\right\}\right\}\right\}$$

$$p^2: \phi_2 = -\mathcal{L}^{-1}\left\{\frac{1}{s}\,\mathcal{L}\left\{L\phi_1 + \widetilde{H}_1\right\}\right\}$$

$$\vdots$$

Hence, the solution of eq.(1) is obtained by taking $p \to 1$, i.e.

$$\phi(x,y,t) = \sum_{n=0}^{\infty} \phi_n$$

3.Condition of convergence of Accelerated HPTM

Here, we emphasize on condition of convergence of the above introduced method.

Theorem 3.1: Let ϕ and ϕ_n be elements of a Banach space, then that the series

$$\phi(x,y,t) = \sum_{n=0}^{\infty} \phi_n p^n$$

converges to the solution of eq.(1) if $\|\phi_{n+1}\| \leq \kappa\|\phi_n\|$, where $0 < \kappa < 1$.This conditions of convergence of the series is proved in [5-6].

4.Application of Accelerated HPTM

Third order KdV equation:The generalized third order KdV equation is given as

$$\frac{\partial \phi}{\partial t} + \phi^p(x,t)\left(\frac{\partial \phi}{\partial x}\right) + \frac{\partial^3 \phi}{\partial x^3} = 0$$

Example 1: Consider the third Order KdV equation [20]:

$$\frac{\partial \phi}{\partial t} + \phi^4(x,t)\left(\frac{\partial \phi}{\partial x}\right) + \frac{\partial^3 \phi}{\partial x^3} = 0\,,\ \phi(x,0) = A^{\frac{1}{4}}\sqrt{\operatorname{sech}[kx]} \tag{8}$$

The closed form of solution is given as $\phi(x,t) = A^{\frac{1}{4}}\sqrt{\operatorname{sech}[kx - ct]}$, where A, k, m and c are constants, $A = \frac{2(p+1)(p+2)}{m^2}k^2$ and $c = \frac{4k^2}{m^2}$. In this case we have chosen $p = 4$.

On applying Laplace transformation to eq. (8), we get

$$\bar{\phi}(x,s) = \frac{1}{s}\phi(x,y.0) - \frac{1}{s}\mathcal{L}\left\{\left(\frac{\partial^3 \phi}{\partial x^3}\right) + \phi^4(x,t)\left(\frac{\partial \phi}{\partial x}\right)\right\} \tag{9}$$

Now operating Inverse Laplace transformation to eq. (9), we have

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$$\phi(x,t) = \phi(x,0) - \mathcal{L}^{-1}\left\{\frac{1}{s}\,\mathcal{L}\left\{\left(\frac{\partial^3\phi}{\partial x^3}\right) + \phi^4(x,t)\left(\frac{\partial\phi}{\partial x}\right)\right\}\right\} \tag{10}$$

Now apply HPM i.e. $\phi(x,t) = \sum_{n=0}^{\infty}\phi_n\,(x,t)p^n$ eq.(10) reduces to

$$\sum_{n=0}^{\infty}\phi_n p^n = \phi(x,0) - \mathcal{L}^{-1}\left\{\frac{1}{s}\,\mathcal{L}\left\{\left(\left(\sum_{n=0}^{\infty}\phi_n\,p^n\right)_{xxx}\right) - \sum_{n=0}^{\infty}\widetilde{H}_n p^n\right\}\right\}, \tag{11}$$

where $\widetilde{H}_n$ is accelerated He's polynomial, where the initial couple of terms of $\widetilde{H}_n$ are given as:

$$\widetilde{H}_0 = \phi_0^4\phi_{0x}$$

$$\widetilde{H}_1 = 4\phi_0^3\phi_{0x}\phi_1 + 6\phi_0^2\phi_{0x}\phi_1^2 + 4\phi_0\phi_{0x}\phi_1^4 + \phi_0^4\phi_{1x} + 4\,\phi_0^3\phi_1\phi_{1x} + 6\,\phi_0^2\phi_1^2\phi_{1x} + 4\phi_0\phi_1^3\phi_{1x} + \phi_1^4\phi_{1x}$$

$$\vdots$$

On equating the term containing same power of p on the both sides of the eq. (11), we get

$$p^0: \phi_0 = \phi(x,0) = A^{\frac{1}{4}}\sqrt{\mathrm{sech}[kx]},$$

$$p^1: \phi_1 = -\mathcal{L}^{-1}\left\{\frac{1}{s}\,\mathcal{L}\{(\phi_{0xxx}) + \widetilde{H}_0\}\right\}$$

$$= \frac{1}{16}A^{1/4}kt(8A - 29k^2 + k^2\cosh[2kx])\mathrm{sech}[kx]^{7/2}\sinh[kx]$$

$$p^2: \phi_2 = -\mathcal{L}^{-1}\left\{\frac{1}{s}\,\mathcal{L}\{(\phi_{1xxxx}) + \widetilde{H}_1\}\right\}$$

$$\begin{aligned}\phi_2 = -\frac{1}{64424509440}A^{1/4}k^2t^2\mathrm{Sech}[kx]^{37/2}(&-4801167360A^2 + 68266598400Ak^2\\ &- 188686502400k^4 + 872939520A^4k^2t^2 - 9438658560A^3k^4t^2\\ &+ 34024181760A^2k^6t^2 - 40889949120Ak^8t^2 - 387973120A^6k^4t^4\\ &+ 7213875200A^5k^6t^4 - 53658501120A^4k^8t^4 + 199581670400A^3k^{10}t^4\\ &- 371205371200A^2k^{12}t^4 + 276190420335Ak^{14}t^4\\ +80\big(&-3667584096k^4 + 7012352A^6k^4t^4 - 130973696A^5k^6t^4\\ &+ 1024A^4k^2t^2(6240 + 955577k^6t^2)\\ &- 1280A^3k^4t^2(52992 + 2855675k^6t^2)\\ &- 2Ak^2(-668608512 + 141448200k^6t^2 + 2550356323k^{12}t^4)\\ &+ A^2(-96104448 + 239958720k^6t^2 + 6827217235k^{12}t^4)\big)\cosh[2kx]\\ -40\big(&3068232192k^4 + 4980736A^6k^4t^4 - 94732288A^5k^6t^4\\ &+ 8192A^4k^2t^2(2784 + 87923k^6t^2)\\ &- 64A^3k^4t^2(3890016 + 42758435k^6t^2)\\ &- Ak^2(1163255808 + 1098647808k^6t^2 + 3943752421k^{12}t^4)\\ &+ A^2(92454912 + 905865984k^6t^2 + 5195650100k^{12}t^4)\big)\cosh[4kx]\end{aligned}$$

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$$+(-659619840A^2+2977628160Ak^2-2020193280k^4-578027520A^4k^2t^2$$
$$+6157762560A^3k^4t^2-21858094080A^2k^6t^2+25852872960Ak^8t^2$$
$$+26214400A^6k^4t^4-532152320A^5k^6t^4+4293795840A^4k^8t^4$$
$$-17228544000A^3k^{10}t^4+34400492880A^2k^{12}t^4$$
$$-27360886560Ak^{14}t^4)\cosh[6kx]$$
$$+(365690880A^2-8875376640Ak^2+28089169920k^4+39321600A^4k^2t^2$$
$$-529367040A^3k^4t^2+2280960000A^2k^6t^2\mathrm{Cosh}[8kx]$$
$$-3185452800Ak^8t^2+6881280A^5k^6t^4-106250240A^4k^8t^4\mathrm{Cosh}[8kx]$$
$$+614302720A^3k^{10}t^4-1576373440A^2k^{12}t^4$$
$$+1515008460Ak^{14}t^4)\cosh[8kx]$$
$$+(286064640A^2-5175705600Ak^2+15372057600k^4+66846720A^4k^2t^2$$
$$-731381760A^3k^4t^2+2660981760A^2k^6t^2-3219621120Ak^8t^2$$
$$+696320A^4k^8t^4-7936000A^3k^{10}t^4+30127600A^2k^{12}t^4$$
$$-38099040Ak^{14}t^4)\cosh[10kx]$$
$$+(80609280A^2-1239613440Ak^2+3512586240k^4+9584640A^3k^4t^2$$
$$-70502400A^2k^6t^2+129484800Ak^8t^2+33280A^3k^{10}t^4$$
$$-251360A^2k^{12}t^4+474360Ak^{14}t^4)\cosh[12kx]$$
$$+(8847360A^2-89456640Ak^2+210896640k^4+414720A^2k^6t^2-1539840Ak^8t^2$$
$$+720A^2k^{12}t^4-2720Ak^{14}t^4)\cosh[14kx]$$
$$+(7987200Ak^2\mathrm{C}-29928960k^4+4800Ak^8t^2+5Ak^{14}t^4)\cosh[16kx]$$
$$+3840k^4\cosh[18kx]$$
$$+(-2988441600A^3kt+21429288960A^2k^3t-38419804160Ak^5t$$
$$+698351616A^5k^3t^3-10190585856A^4k^5t^3+55772872704A^3k^7t^3$$
$$-135684965376A^2k^9t^3+123805192224Ak^{11}t^3)\sinh[2kx]$$
$$+(-2933391360A^3kt+20875837440A^2k^3t-37141934080Ak^5t$$
$$-132120576A^5k^3t^3+2032140288A^4k^5t^3-11690237952A^3k^7t^3$$
$$+29818257408A^2k^9t^3-28460631072Ak^{11}t^3)\sinh[4kx]$$
$$+(-1064304640A^3kt+7373127680A^2k^3t-12756725760Ak^5t-232783872A^5k^3t^3$$
$$+3374579712A^4k^5t^3-18339545088A^3k^7t^3+44283632640A^2k^9t^3$$
$$-40086192864Ak^{11}t^3)\sinh[6kx]$$
$$+(+73400320A^3kt-686817280A^2k^3t+1513287680Ak^5t+66060288A^5k^3t^3$$
$$-1016070144A^4k^5t^3+5843681280A^3k^7t^3-14898364416A^2k^9t^3$$
$$+14210181408Ak^{11}t^3)\sinh[8kx]$$
$$+(162529280A^3kt-1197670400A^2k^3t+2199654400Ak^5t+13369344A^4k^5t^3$$
$$-150847488A^3k^7t^3+566774784A^2k^9t^3$$
$$-709177440Ak^{11}t^3)\sinh[10kx]$$
$$+(34078720A^3kt-237240320A^2k^3t\mathrm{S}+410603520Ak^5t+958464A^3k^7t^3$$
$$-7176192A^2k^9t^3+13422432Ak^{11}t^3)\sinh[12kx]$$
$$+(2949120A^2k^3t-10711040Ak^5t+27648A^2k^9t^3-103776Ak^{11}t^3)\sinh[14kx]$$
$$+(51200Ak^5t+240Ak^{11}t^3)\sinh[16kx])$$
$$\vdots$$

The solution of eq.(8) is given by $\phi(x,t)=\sum_{n=0}^{\infty}\phi_n p^n$ as $p\to 1$ i.e.

$$\phi(x,y,t)=\phi_0+\phi_1+\phi_2+\cdots$$

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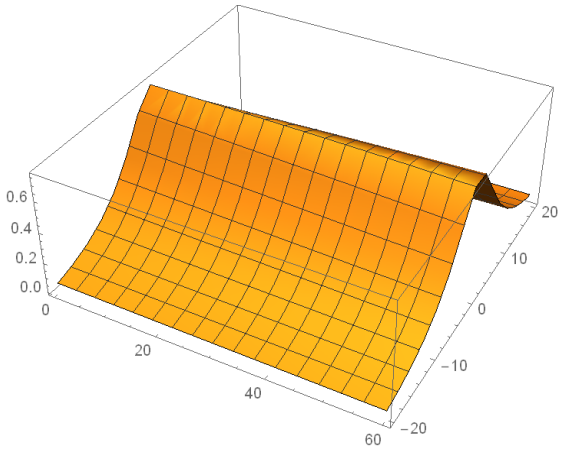


Fig.1 : Approx. $\phi(x,t)$,
when $0 \le t \le 60$ and $-20 \le x \le 20$)
$A = 0.3375$ and $k = 0.3$

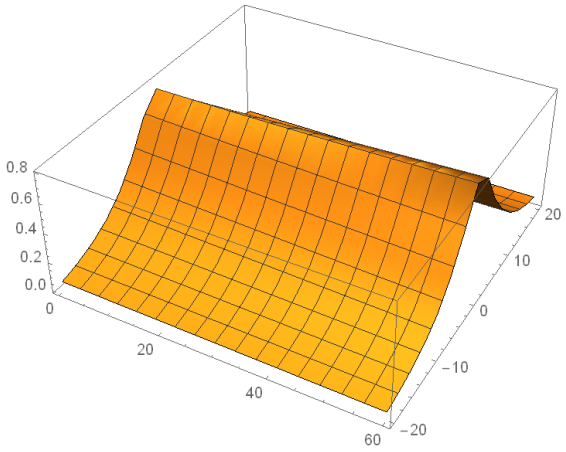


Fig.2: Exact $\phi(x,t)$,
(when $0 \le t \le 60$ and $-20 \le x \le 20$)
$A = 0.3375$, $k = 0.3$ and $c = 0.00675$

Table1: The computed value of $\phi(x,t)$ when $= 0.3375$, $k = 0.3$ and $c = 0.00675$

x	t	$Acc.HPTM\ \phi(x,t)$	$Exact\ \phi(x,t)$
0	0	0.7621991222319221	0.7621991222319221
0	1	0.7621904404229183	0.7622078041562987
0	2	0.7621643963802982	0.7622338499294288
0	3	0.7621209942566095	0.7622772595513122
1	0	0.7454868179223733	0.7454868179223733
1	1	0.7462123374326297	0.7462274238746495
1	2	0.7469229304262485	0.7469843386248235
1	3	0.7476184932468035	0.7477590560061192
2	0	0.700041902121695	0.700041902121695
2	1	0.7013062102326961	0.7013154266166637
2	2	0.7025613302006107	0.7025988714125879
2	3	0.7038071220712073	0.7038931172919621
3	0	0.6366962019740304	0.6366962019740304
3	1	0.6382337291908284	0.638237036515011
3	2	0.6397678037619896	0.6397808969584535
3	3	0.6412983136775822	0.6413274688998503

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Fouth order KdV equation: The generalized fourth order KdV equation is given as

$$\frac{\partial \psi}{\partial t} + (p+1)\psi^p(x,t)\left(\frac{\partial \psi}{\partial x}\right) + \frac{\partial^4 \psi}{\partial x^4} = 0.$$

Example 2: Consider the fourth order KdV equation [20]:

$$\frac{\partial \psi}{\partial t} + 5\psi^4(x,t)\left(\frac{\partial \psi}{\partial x}\right) + \frac{\partial^4 \psi}{\partial x^4} = 0\ ,\ \psi(x,0) = A\sqrt{\text{sech}[kx]} \tag{12}$$

Closed form of solution of this equation is given by $\psi(x,t) = A\sqrt{\text{sech}[kx - ct]}$, where A, k, m and c are constants i.e. $A = \frac{2(p+1)(p+2)}{m^2}k^2$ and $c = \frac{4k^2}{m^2}$.

On applying Laplace transformation to eq. (8), we have

$$\bar{\psi}(x,s) = \frac{1}{s}\psi(x,y.0) - \frac{1}{s}\mathcal{L}\left\{\left(\frac{\partial^4 \psi}{\partial x^4}\right) + 5\psi^4(x,t)\left(\frac{\partial \psi}{\partial x}\right)\right\} \tag{13}$$

Now operating Inverse Laplace transformation to eq. (9), we have

$$\psi(x,t) = \psi(x,0) - \mathcal{L}^{-1}\left\{\frac{1}{s}\,\mathcal{L}\left\{\left(\frac{\partial^4 \psi}{\partial x^4}\right) + 5\psi^4(x,t)\left(\frac{\partial \psi}{\partial x}\right)\right\}\right\} \tag{14}$$

Now apply HPM i.e. $\psi(x,t) = \sum_{n=0}^{\infty}\psi_n\,(x,t)p^n$ eq.(10) reduces to

$$\sum_{n=0}^{\infty}\psi_n p^n = \psi(x,0) - \mathcal{L}^{-1}\left\{\frac{1}{s}\,\mathcal{L}\left\{\left(\left(\sum_{n=0}^{\infty}\psi_n\,p^n\right)_{xxxx}\right) + \sum_{n=0}^{\infty}\widetilde{H}_{1n}p^n\right\}\right\}, \tag{15}$$

where $\widetilde{H}_{1n}$ is accelerated He's polynomial, where the initial couple of terms of $\widetilde{H}_{1n}$ are given as:

$$\widetilde{H}_{10} = 5\psi_0^4\psi_{0x},$$

$$\widetilde{H}_{11} = 20\psi_0^3\psi_{0x}\psi_1 + 30\psi_0^2\psi_{0x}\psi_1^2 + 20\psi_0\psi_{0x}\psi_1^4 + 5\psi_0^4\psi_{1x} + 20\,\psi_0^3\psi_1\psi_{1x} + 30\,\psi_0^2\psi_1^2\psi_{1x} + 20\psi_0\psi_1^3\psi_{1x} + 5\psi_1^4\psi_{1x}$$

$$\vdots$$

On equating the term containing same power of p on the both sides of the eq. (11), we get

$$p^0: \psi_0 = \psi(x,0) = A\sqrt{\text{sech}[kx]},$$

$$p^1: \psi\psi_1 = -\mathcal{L}^{-1}\left\{\frac{1}{s}\,\mathcal{L}\{(\psi_{0xxxx}) + \widetilde{H}_{10}\}\right\}$$

$$= -\frac{1}{128}Akt\,(\text{sech}[kx])^{9/2}(531k^3 - 308k^3\cosh[2kx] + k^3\cosh[4kx] - 160A^4\sinh[2kx])$$

$$p^2: \psi_2 = -\mathcal{L}^{-1}\left\{\frac{1}{s}\,\mathcal{L}\{(\phi_{1xxxx}) + \widetilde{H}_{11}\}\right\}$$

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$$\psi_2 = \frac{1}{2199023255552} Ak^2 t\,(\mathrm{sech}[kx])^{47/2}(-3136\big(-165043523008k^6 + 16384000000A^{24}k^4t^4 - 1920000A^{16}k^2t^2(8448 + 132128k^4t + 1188257k^8t^2) + 25A^8(159318016 + 2289025024k^4t + 17262426624k^8t^2 + 108377311168k^{12}t^3 + 963602106943k^{16}t^4)\big)\cosh[kx] + 64\big(4863026756672k^6 + 671744000000A^{24}k^4t^4 - 3200000A^{16}k^2t^2(-30976 + 1519072k^4t + 43093463k^8t^2) + 25A^8(-5749932032 - 55249485824k^4t - 60036478464k^8t^2 + 3467327894464k^{12}t^3 + 85193975797735k^{16}t^4)\big)\cosh[3kx] + \cosh[5kx](-4773747097600A^8 + 71489784524800k^6 + 5276067430400A^8k^4t - 27567063040000A^{16}k^2t^2 + 642187223040000A^8k^8t^2 - 576628981760000A^{16}k^6t^3 + 4576813931110400A^8k^{12}t^3 + 29884416000000A^{24}k^4t^4 - 414905344000000A^{16}k^{10}t^4 - 81808084751344000A^8k^{16}t^4) + \cosh[7kx](-1472410419200A^8 - 53001547788288k^6 + 37933652377600A^8k^4t - 24274534400000A^{16}k^2t^2 + 361919835340800A^8k^8t^2 - 49968578560000A^{16}k^6t^3 - 1822064070041600A^8k^{12}t^3 - 17301504000000A^{24}k^4t^4 + 30191119974400000A^{16}k^{10}t^4 + 24090287406908800A^8k^{16}t^4) + \cosh[9kx](-3565158400A^8 - 60358485792768k^6 + 24903837286400A^8k^4t - 7308574720000A^{16}k^2t^2 - 6398140416000A^8k^8t^2 + 128564592640000A^{16}k^6t^3 - 7020799780864000A^8k^{12}t^3 - -6815744000000A^{24}k^4t^4 - 1033824870400000A^{16}k^{10}t^4 - 35128863070240000A^8k^{16}t^4) + \cosh[11kx](260256563200A^8 - 27999819982848k^6 + 7227886796800A^8k^4t + 765460480000A^{16}k^2t^2 - 497723326464000A^8k^8t^2 + 22021406720000A^{16}k^6t^3 + 3402224527360000A^8k^{12}t^3 + 2621440000000A^{24}k^4t^4 + 1010536038400000A^{16}k^{10}t^4 + 210977173840000A^8k^{16}t^4) + \cosh[13kx](143497625600A^8 - 6529278343168k^6 + 427832115200A^8k^4t + 9961472000000A^{16}k^2t^2 - 5563513651200A^8k^8t^2 - 8463319040000A^{16}k^6t^3 - 333555645952000A^8k^{12}t^3 - 532848640000A^{16}k^{10}t^4 - 2062888050400A^8k^{16}t^4) + \cosh[15kx](41471180800A^8 - 379814758400k^6 - 273691443200A^8k^4t + 178257920000A^{16}k^2t^2 + 1981188096000A^8k^8t^2 + 22282240000A^{16}k^6t^3 + 2508963328000A^8k^{12}t^3 + 696320000A^{16}k^{10}t^4 + 7874552800A^8k^{16}t^4) + \cosh[17kx](6658457600A^8 + 149093439488k^6 - 51498188800A^8k^4t - 9811968000A^8k^8t^2 - 616499200A^8k^{12}t^3 - 12877600A^8k^{16}t^4) + \cosh[19kx](471859200A^8 + 23660423168k^6 + 1179648000A^8k^4t + 11059200A^8k^8t^2 + 460800A^8k^{12}t^3 + 7200A^8k^{16}t^4) - 399951872k^6\cosh[21kx] + 2048k^6\cosh[23kx] + \sinh[kx](-342138421248000A^4k^3 + 11096031232000A^{12}kt - 82868690452480A^4k^7t + 2409592258560000A^{12}k^5t^2 - 757114111252480A^4k^{11}t^2 - 45508198400000A^{20}k^3t^3 + 31531221073920000A^{12}k^9t^3 - 7639711671220160A^4k^{15}t^3 - 10373169152000000A^{20}k^7t^4 + 4828783959193600A^{12}k^{13}t^4 - 32253358563829942A^4k^{19}t^4)$$

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$$
\begin{aligned}
&+\sinh[3kx](-79894645309440A^4k^3 + 24591204352000A^{12}kt \\
&\quad - 1716324610048000A^4k^7t + 430732738560000A^{12}k^5t^2 \\
&\quad - 1182223106329600A^4k^{11}t^2 - 64434995200000A^{20}k^3t^3 \\
&\quad + 3542860185600000A^{12}k^9t^3 - 5357215289709920A^4k^{15}t^3 \\
&\quad - 6537773056000000A^{20}k^7t^4 + 4604270261248000A^{12}k^{13}t^4 \\
&\quad + 193436973021793970A^4k^{19}t^4) \\
&+\sinh[5kx](-7971473326080 0A^4k^3 + 21755854848000A^{12}kt \\
&\quad - 1290725756928 00A^4k^7t + 2007076700160 00A^{12}k^5t^2 \\
&\quad - 3139236304742 40A^4k^{11}t^2 - 2883584000000A^{20}k^3t^3 \\
&\quad - 8696345886720 00A^{12}k^9t^3 + 4242626120298720A^4k^{15}t^3 \\
&\quad + 6867681280000 00A^{20}k^7t^4 - 2688303367680 0000A^{12}k^{13}t^4 \\
&\quad - 6969925619938 4490A^4k^{19}t^4) \\
&+\sinh[7kx](-4989390422016 0A^4k^3 + 10687086592000A^{12}kt \\
&\quad - 4321934868480 0A^4k^7t - 39382941696000A^{12}k^5t^2 \\
&\quad + 2256848939212 80A^4k^{11}t^2 + 27158118400000A^{20}k^3t^3 \\
&\quad - 1337715326976 000A^{12}k^9t^3 + 7151109832800 00A^4k^{15}t^3 \\
&\quad + 1115619328000 00A^{20}k^7t^4 + 1380662486425 6000A^{12}k^{13}t^4 \\
&\quad + 1476009579046 9080A^4k^{19}t^4) \\
&+\sinh[9kx](-2005236449280 0A^4k^3 + 2290089984000A^{12}kt + 188156510208 0A^4k^7t \\
&\quad - 7042026700800 0A^{12}k^5t^2 + 1066537976832 00A^4k^{11}t^2 \\
&\quad + 1059061760000 0A^{20}k^3t^3 + 1096120074240 00A^{12}k^9t^3 \\
&\quad - 9970602618412 80A^4k^{15}t^3 - 1661403136000 00A^{20}k^7t^4 \\
&\quad - 2787708628992 000A^{12}k^{13}t^4 - 1705845504533 400A^4k^{19}t^4) \\
&+\sinh[11kx](-442540032000 0A^4k^3 - 50751078400 0A^{12}kt + 64224710246 40A^4k^7t \\
&\quad - 1974809395200 0A^{12}k^5t^2 - 1805087346432 0A^4k^{11}t^2 \\
&\quad - 1625292800000 0A^{20}k^3t^3 + 1628839772160 00A^{12}k^9t^3 \\
&\quad + 2307441661812 00A^4k^{15}t^3 + 2545745920000 0A^{20}k^7t^4 \\
&\quad + 2046337648640 00A^{12}k^{13}t^4 + 8840527017310 5A^4k^{19}t^4) \\
&+\sinh[13kx](-810221568 0A^4k^3 - 49073356800 0A^{12}kt + 16368187392 00A^4k^7t \\
&\quad + 1375469568000A^{12}k^5t^2 - 8882368688640A^4k^{11}t^2 \\
&\quad - 1101004800000A^{20}k^3t^3 - 2494174003200 0A^{12}k^9t^3 \\
&\quad - 1704904837608 0A^4k^{15}t^3 - 6881280000 0A^{20}k^7t^4 \\
&\quad - 1564351488000A^{12}k^{13}t^4 - 1030467034485A^4k^{19}t^4) \\
&+\sinh[15kx](29309929920 0A^4k^3 - 13316915200 0A^{12}kt - 4351713280 0A^4k^7t \\
&\quad + 1017839616000A^{12}k^5t^2 + 1313229685760A^4k^{11}t^2 \\
&\quad + 128790528000A^{12}k^9t^3 + 164679010720A^4k^{15}t^3 \\
&\quad + 4040960000A^{12}k^{13}t^4 + 5151772010A^4k^{19}t^4) \\
&+\sinh[17kx](7495876608 0A^4k^3 - 1363148800 0A^{12}kt - 4974714880 0A^4k^7t \\
&\quad - 2555904000A^{12}k^5t^2 - 9522119680A^4k^{11}t^2 - 159744000A^{12}k^9t^3 \\
&\quad - 599207840A^4k^{15}t^3 - 3328000A^{12}k^{13}t^4 - 12526090A^4k^{19}t^4) \\
&+\sinh[19kx](5750784000A^4k^3 + 226140160A^4k^7t + 21322240A^4k^{11}t^2 \\
&\quad + 890960A^4k^{15}t^3 + 13945A^4k^{19}t^4) + \sinh[21kx](-206438400A^4k^3 \\
&\quad - 204800A^4k^7t - 12800A^4k^{11}t^2 - 400A^4k^{15}t^3 - 5A^4k^{19}t^4)
\end{aligned}
$$

$$\vdots$$

The solution of eq.(8) is given by $\psi(x,t) = \sum_{n=0}^{\infty} \psi_n p^n$ as $p \to 1$ i.e.

$$\psi(x,y,t) = \psi_0 + \psi_1 + \psi_2 + \cdots$$

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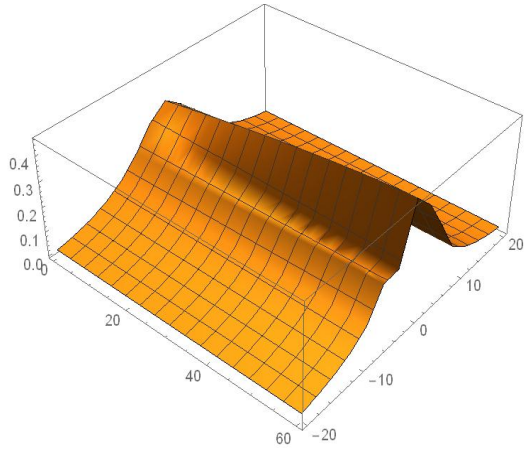


Fig.3 : Approx. $\psi(x,t)$,
when $0 \le t \le 60$ and $-20 \le x \le 20$)
$A = 0.3375$ and $k = 0.3$

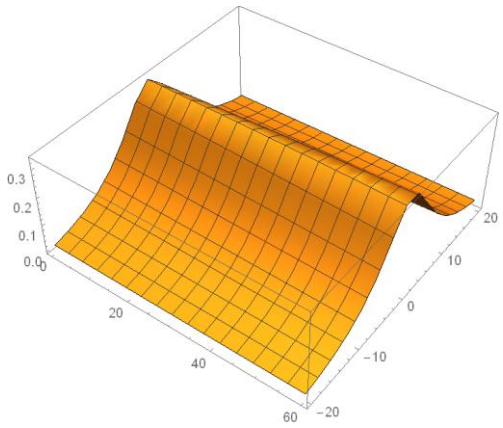


Fig.4: Exact $\psi(x,t)$,
(when $0 \le t \le 60$ and $-20 \le x \le 20$)
$A = 0.3375$, $k = 0.3$ and $c = 0.00675$

Table2: The computed value of $\psi(x,t)$ when $= 0.3375$, $k = 0.3$ and $c = 0.00675$

x	t	Approx.$\psi(x,t)$	Exact $\psi(x,t)$
0	0	0.3375	0.3375
0	1	0.3402227368080811	0.3374961557151493
0	2	0.3429377528101018	0.3374846234736026
0	3	0.3456445590601601	0.3374654051140983
1	0	0.3300998304905467	0.3300998304905467
1	1	0.3266697297056239	0.3304210888436061
1	2	0.3232275346847781	0.3307357377697871
1	3	0.3197729105029733	0.331043731370265
2	0	0.3099769273864128	0.3099769273864128
2	1	0.3067510450213727	0.3105367600797559
2	2	0.3035289513794303	0.3110925242846932
2	3	0.3003105684374207	0.3116441580298164
3	0	0.2819275985742345	0.2819275985742345
3	1	0.2842205067704839	0.2826084120526728
3	2	0.2865129025375428	0.2832876967076470
3	3	0.2888047835614346	0.2839654029414195

5.Conclusion

In this paper, we have applied accelerated HPTM for the series solution of KdV equations of order three and four. In these equations, the nonlinear term $\phi^4(x,t)\left(\frac{\partial \phi}{\partial x}\right)$ and

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$\left(5\psi^4(x,t)\left(\frac{\partial\psi}{\partial x}\right)\right)$ are handled with accelerated He's polynomial. In Table 1 and table 2, we have compared the approximate solution acquired from Accelerated HPTM upto second order with the exact solution. We have used mathematica software 11.1 for plotting the surface graph and calculated tabulated value.. Fig. 1,3 and 2,4 represents the surface graph obtained from the computed value of $\phi(x,t)$ and $\psi(x,t)$ and the exact value of $\phi(x,t)$ and $\psi(x,t)$ for constants $A = 0.3375\,, k = 0.3$ and $c = 0.00675$.As the solution obtained from the proposed technique is close to the exact solution. Hence, we conclude that the accelerated HPTM technique is reliable and efficient technique for solving such typical nonlinear PDEs.

References

[1] J. H. He. "A new approach to nonlinear partial differential equations. Communications in nonlinear science and numerical simulation," vol.2, no.4, pp.230-235, Dec. 1997.

[2] J. H. He. "Homotopy perturbation technique." Computer methods in applied mechanics and engineering, vol. 178, no.3, pp.257-262, Aug.1999.

[3] I. L. E. Kalla. "An accelerated homotopy perturbation method for solving nonlinear equations." Journal of Fractional Calculus and Applications,vol. 3(S) no.16, pp.1-6, July 2012.

[4] D. Sharma, P. Singh and S. Chauhan, "Homotopy perturbation transform method with He's polynomial for solution of coupled nonlinear partial differential equations," Nonlinear engineering, vol. 5, no.1, pp.17-23, Jan. 2016.

[5] P. Singh and D. Sharma, "On the problem of convergence of series solution of non-linear fractional partial differential equation," In AIP conference proceedings, vol.1860, no.1, pp. 020027. AIP Publishing, July 2017.

[6] P. Singh and D. Sharma, "Convergence and error analysis of series solution of nonlinear partial differential equation. Nonlinear engineering", vol.7, no.4, pp.303-308, Dec. 2018.

[7] Y. Khan and Q. Wu, "Homotopy perturbation transform method for nonlinear equations using He's polynomials". Computers & mathematics with applications, vol.61, no.8, pp.1963-1967, Apr. 2011.

[8] Y. Khan, N. Faraz, S. Kumar and A. Yildirim, "A coupling method of homotopy perturbation and Laplace transformation for fractional models. University" Politehnica" of Bucharest Scienti c Bulletin, Series A: Applied mathematics and physics, vol.74 no.1, pp.57-68, Jan. 2012.

[9] A. M. Wazwaz, "Construction of solitary wave solutions and rational solutions for the KdV equation by Adomian decomposition method", Chaos, Solitons & Fractals. vol.12, no.12, pp. 2283-93, Sept.2001.

[10] D. Kaya and M. Aassila, "An application for a generalized KdV equation by the decomposition method",Physics Letters A, vol.299, no.2-3, pp.201-206, July 2002.

[11] D. Kaya and S.M. El-Sayed, "On a generalized fifth order KdV equations", Physics Letters A, vol.310, no.1, pp.44-51, April 2003.

 Copyright ⓒ 2019Authors

ISSN:0971-1260
Vol-22-Issue-37-December -2019

[12] D. Kaya, "An application for the higher order modified KdV equation by decomposition method", Communications in nonlinear science and numerical simulation, vol.10, no.6, pp.693-702, Sept. 2005.

[13] A. Yildirim,"The homotopy perturbation method for solving the modified Korteweg-de Vries equation", Zeitschrift für Naturforschung A, vol.63, no.10-11, pp.621-626 No. 2008.

[14] M. Kazeminia, S. Soleimani-Amiri and S. A. Zahedi, 2010. "Exact and numerical solutions for nonlinear higher order modified KdV equations by using variational iteration method", advanced studies in theoretical physics, vol.4, no.9, pp.437-447, 2010.

[15] D. Sharma, P. Singh and S. Chauhan, " Solution of fifth-order Korteweg and de Vries equation by homotopy perturbation transform method using He's polynomial" Nonlinear engineering, vol.6, no.2, pp.89-93, June 2017.

[16] D. Sharma, P. Singh and S. Chauhan., 2016. "Homotopy perturbation Sumudu transform method with He's polynomial for solutions of some fractional nonlinear partial differential equations", International journal of nonlinear science, vol.21, no.2, pp.91-97, April 2016.

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